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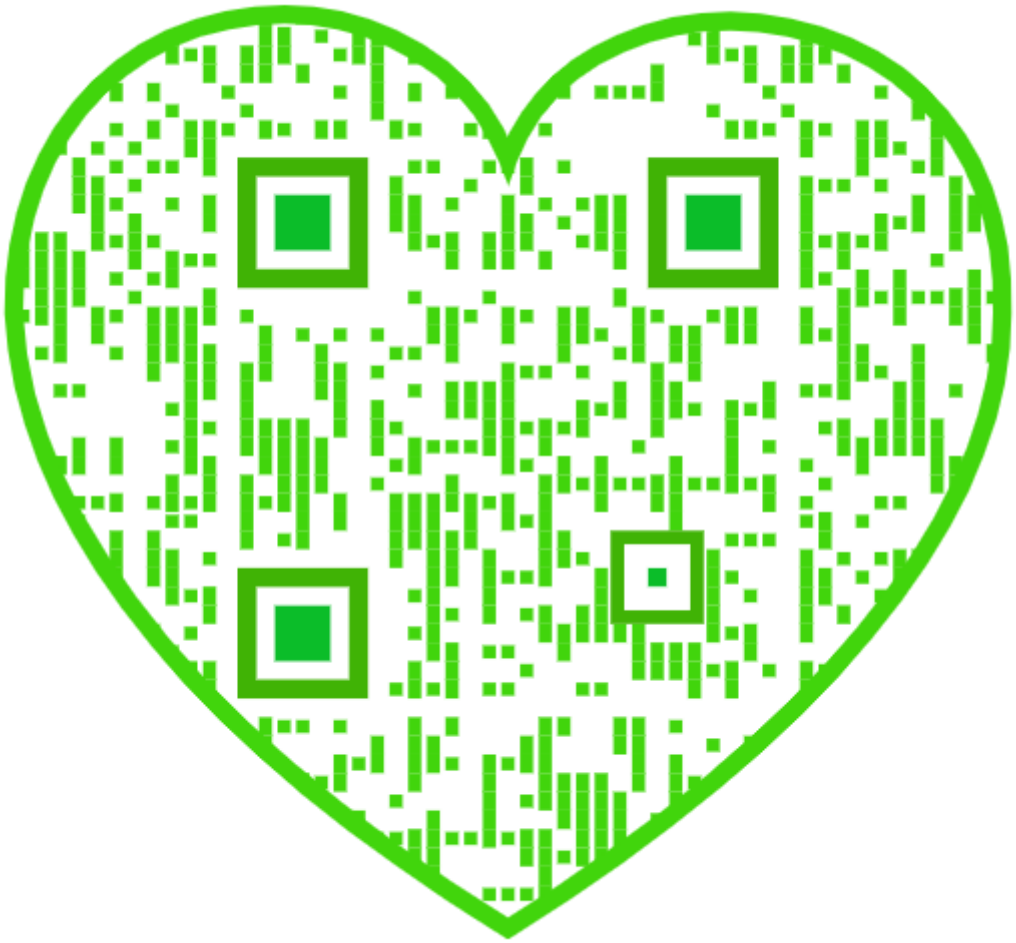

THE

MECHANIC'S ASSISTANT:

A THOROUGH PRACTICAL TREATISE ON

MENSURATION

AND

THE SLIDING RULE:

TEACHING THE MANNER OF DRAWING ALL REGULAR SUPERFICIES, AND THE MOST CONCISE METHODS OF FINDING THE AREAS OF ALL REGULAR SUPERFICIES, AND THE CONTENTS OF ALL REGULAR SOLIDS,

BOTH BY NUMBERS AND BY THE SLIDING RULE.

TREATING ALSO OF

THE LAWS OF MOTION—THE DESCENT OF FALLING BODIES—THE STRENGTH OF MATERIALS—THE MECHANICAL POWERS—THE ELASTICITY AND FORCE OF STEAM—SPECIFIC GRAVITIES—LEVELLING—THE PENDULUM, ETC.

ADAPTED FOR THE USE OF

CARPENTERS, SHIPWRIGHTS, WHEELWRIGHTS, SAWYERS, GAUGERS, LUMBERMEN, STUDENTS, AND ARTISANS GENERALLY.

BY D. M. KNAPEN, A. M.

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PREFACE.

THE leading aim of the author, in preparing the present treatise, has been to produce a work of a more *practical* nature, and better adapted to the business of life, than any similar work hitherto offered to the public. Though the work is designed to be useful to all classes, embracing a greater variety of topics, and occupying a wider range than most treatises on mensuration, yet it is more particularly designed to meet the wants of the mechanical community, and of those whose professions involve construction, measurement, and the use of the *sliding*, or *carpenter's rule*.

As the work is *strictly practical*, very little matter of an abstruse nature has been introduced, and the algebraical formulas on which some of the rules are founded, have been omitted.

Though the author believes the present treatise contains more original matter than almost any other volume on a similar subject, yet he would cheerfully acknowledge his indebtedness to Bell's excellent treatise on Practical Mathematics, to Nicholson's British Mechanics, to Brande's Encyclopedia of Science, and to several other good works, from which he has borrowed a portion of his materials.

It is hoped that the work will meet with a favorable reception from the public, and that, while it answers the purpose of a good practical treatise, it will serve to inspire those into whose hands it may come, with a desire to acquire a thorough knowledge of the principles on which the more abstruse rules in this volume are founded; by which means they may be pre-

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pared, not only to improve and abbreviate known rules, but to deduce new ones for themselves.

Great pains have been taken to render the work accurate, concise, and easy to be understood; and, though it is far from being voluminous, it will, it is believed, be found to contain all the most useful and essential principles of that important branch of analysis, the mensuration of superficies and solids, the general principles involved in mechanics, and the construction of machinery, together with other matters of practical importance.

The carpenter's sliding rule is fully explained, and is applied to the mensuration of all regular superficies and solids, to finding the weight of bodies, to levelling, and to many other purposes.

The machinist, the carpenter, the lumberman, the millwright, the shipwright, the gauger, the cooper, the painter, and the engineer, will each find the work to be admirably adapted to supply his wants in the every-day business of life; and the student will find, in this volume, many things calculated to amuse and instruct.

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GAUGE POINTS—¶ 14, 15, 19, 21, 22, 33, 34, 36, 82.

Gauge Points for *capacities*, bushels, gallons, &c., ¶ 58; for the weight of bodies, ¶ 69; for superficies, solids, weight of bodies, &c., ¶ 82.

Gauge Points denoted by *dots* on the rule, ¶ 22, page 77.

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Timber Measurement, ¶ 33.

Steam-Engine, &c., ¶ 79, page 245.

Friction, ¶ 79, page 246.

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SIGNS AND DEFINITIONS.

The following signs are occasionally used in this Treatise:—

This (+) is the sign of *addition*, called *plus*, and signifies that the numbers before and after it are to be added together.

This (−) is the sign of *subtraction*, called *minus*, and signifies that the number following it is to be deducted from the number preceding it.

This (×) is the sign of *multiplication*, and signifies that the numbers on each side of it are to be multiplied into each other.

This (÷) is the sign of *division*, and signifies that the number preceding it is to be divided by the number following it.

This (=) is the sign of *equality*, and signifies that the number or numbers before it equal the number or numbers following it. Thus, $8 = 8$; $8 + 4 = 12$; $9 - 3 = 6$; $45 \div 5 = 9$; $(6 \times 3) \div 2 = 9$; and $(12-4) \times 3 \div 6 = 4$. When numbers are included between the sign of a parenthesis, as (4+6), it signifies that the numbers thus included are to be taken together, and their sum or difference is to be added to, or multiplied, or divided by the number following or preceding, according as the sign following or preceding indicates.

This (:) and this (: :) are the signs of proportion. Thus, 3 : 6 :: 4 : 8; that is, as 3 is to 6, so is 4 to 8.

This ($\sqrt{9}$) is the sign of the square root, and signifies that the square root of the number following is to be extracted. Thus, $\sqrt{9} = 3$; and $\sqrt{25} = 5$.

A *mean* proportional between two numbers is the square root of their product; and a *third* proportional to two numbers is the quotient obtained by dividing the square of one of the numbers by the other.

The *rectangle* of two numbers signifies their product.

The word *apothegm* is used to signify the distance from the centre of any figure to the centre of one of its sides; it is likewise used to signify the perpendicular height of a triangle.

An *abscise* is the segment, or part of a line. See ¶ 29 and 30.

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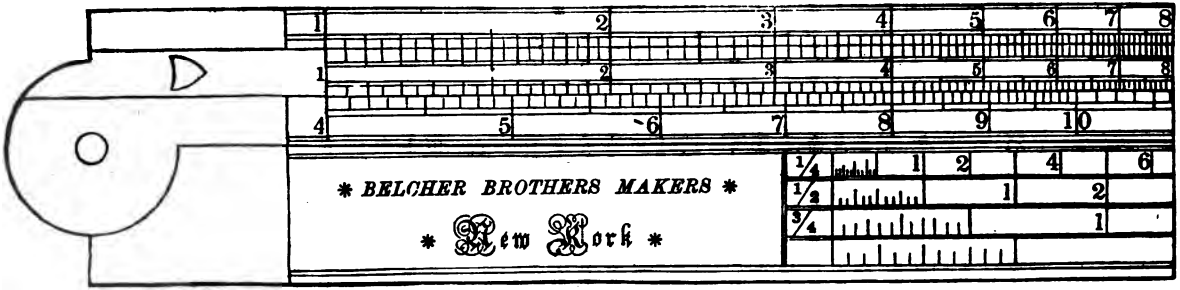
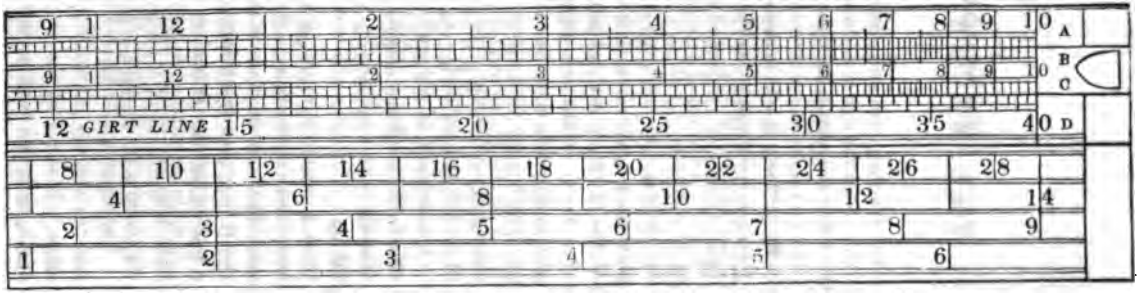
SLIDING RULE.





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THE MECHANIC'S ASSISTANT.

DESCRIPTION OF THE SLIDING RULE.

¶ 1. This useful instrument was first invented by Thomas Everard, an Englishman, in the year 1683, and was made by Isaac Carver, a celebrated mechanic who resided in the vicinity of London. When shut, the rule is one foot in length, but by opening and drawing out the slider, it may be extended to three feet. One side of the rule, together with the under side of the slider, is divided into inches and eighths, or sixteenths of an inch; whilst the edge of the rule is divided into tenths and hundredths of a foot. By this means inches and eighths of an inch may be reduced to tenths and hundredths of a foot, simply by examining the edge of the rule opposite any number of inches and eighths.

On the face of the rule over the slider is a line of numbers called the A line. (This line was invented by Mr. Edmund Gunter, who applied the logarithms of numbers to a rule, by taking the lengths expressed by the figures in those logarithms from a scale of equal parts, and applying them to lines, as laid down on the rule.) On the slider there are two lines exactly like the former, or A line, the upper line being called the B line, and the lower one the C line. Under the slider is a line called the D line, or *girt* line. And at the right hand, near the end of the rule and opposite the lines which they represent, will be found the letters A, B, C, and D.

The numbers on these four lines increase from left to right; and the manner of *reading off* on the lines A, B, and C is the same. If the 1 at the beginning of the line A, be called one-tenth, then the next primary division towards the right, marked

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2, will represent two-tenths, and the 3 will represent three-tenths, the 4 four-tenths, the 5 five-tenths, the 6 six-tenths, the 7 seven-tenths, the 8 eight-tenths, the 9 nine-tenths, and the 1 at the middle of the rule will be ten-tenths, or one; and the following 12 will be twelve-tenths, or one and two-tenths; and the following 2 will count two, the 3 will count three, the 4 four, the 5 five, the 6 six; and so on to the end of the line, the 10 at the end of the line representing ten. Again, if the 1 at the beginning of the lines A, B, or C, represents one, or unity, the 2 following will count two, the 3 three, the 4 four, the 5 five; the 1 at the middle of the line will represent ten, the following 2 will be twenty, the 3 will stand for thirty, the 4 for forty, the 5 for fifty, the 6 for sixty, and so on, the 10 at the right representing one hundred. If we begin again at the left, and call the 1 ten, the following 2 will represent twenty, the 3 will stand for thirty, the 4 for forty, the middle 1 for one hundred, the following 2 for two hundred, the 3 for three hundred, the 4 for four hundred, and the 1 at the end, or the 10, will stand for one thousand.

Thus any value, as, for example, ten, or one hundred, or one thousand, or ten thousand, may be put on the 1 at the beginning of the line, or on the 1 at the middle of the rule; and then the numbers will increase towards the right, (as illustrated above,) but will diminish to the left in the same ratio. If, for example, the 10 at A be called ten thousand, the 9 to the left will represent nine thousand, the 8 will stand for eight thousand, the 7 for seven thousand, the 1 at the middle of the line for one thousand, the 9 for nine hundred, the 8 for eight hundred, and the 1 at the left for one hundred. Or, if we call the 10 at A, one, the 9 to the left will be nine-tenths, the 8 will stand for eight-tenths, the 7 for seven-tenths, the 1 at the middle of the line for one-tenth, the 9 to the left for nine-hundredths, the 8 for eight-hundredths, and so on; the 1 at the left representing one-hundredth, or $\frac{1}{100}$.

It is manifest that the value of the subdivisions must depend on that of the primary divisions; that is, the intermediate spaces must have a proportional value. If, for example, the 1 at the left hand be called one unit, and the 2 on the right

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two units, then, as there are ten long marks between 1 and 2 to denote the principal subdivisions, each of these marks must be called one-tenth. Thus, if we call the 1 one unit, the first long mark between 1 and 2 will represent one and one-tenth, or 1.1, and the second one and two-tenths, or 1.2, and the third 1.3, the fourth 1.4, the fifth 1.5, or one and a half, and so on to the next primary division, which represents two units. Then between 2 and 3 there are ten marks, or if more than ten there will be ten long marks to denote the principal subdivisions, and these must be reckoned as before, the first mark (or first long mark) after the 2 representing 2.1, the second 2.2, the third 2.3, and so on to 3; and in like manner we continue to reckon till we come to 1 or 10 at the middle of the rule; unless (as is sometimes the case) there should be but five spaces or divisions laid down on the rule between 8 and 9, and 9 and 10, in which case you must count these spaces two-tenths, the first after the 9 representing nine and two-tenths, or 9.2, the second 9.4, the third 9.6, and so on to 1, or 10 at the middle of the line. Then, as there are ten long marks, or ten principal subdivisions between 10 and 20, each of these must be reckoned one unit; the first, therefore, will represent 11, the second 12, the third 13, the fourth 14, and so on. Or, beginning at the left hand at 1, and calling the 1 one-tenth, and the 2 following two-tenths, then, as there are ten principal divisions between 1 and 2, or one-tenth and two-tenths, each of these must be reckoned one-tenth of one-tenth, or one-hundredth. Calling the 1 one-tenth, or .1, the first long mark will represent .11, or eleven hundredths, the second .12, the third .13, the fourth .14, and so on to .2. And between .2 and .3, each mark, or (if there be more than ten spaces or divisions) each long mark must be counted as before, one-tenth of one-tenth, or one-hundredth. Between .3 and .4, between .4 and .5, and so on to .8, there are ten marks or divisions between each of the principal divisions; and consequently each of these minor divisions must be valued as before, being called one-hundredth. Between .8 and .9, and .9 and .10, if there be ten spaces, each will be equal to one-hundredth; but if there be but five divisions, each space must

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be reckoned or called two-hundredths. For example, calling the 9 nine-tenths, or .9, if there be but five spaces between .9 and 1, the first mark will be .92, or ninety-two-hundredths, the second .94, the third .96, the fourth .98, and the fifth one-hundred-hundredths, or one.

Between the 1 at the middle of the rule, and the 2 following, there are ten principal subdivisions, distinguished by the length of the marks, and consequently each of these must be called one-tenth, or .1; and if there be ten more minor subdivisions between 1 and 2, each of these must be reckoned half of one-tenth, or five-hundredths. The reading, therefore, or reckoning, between 1 and 2, will be 1.1, and 1.2, and 1.3, and so on; or, if we compute the minor subdivisions, the *reading* will be 1.05, and 1.1, and 1.15, and 1.20, and 1.25, and 1.30, and 1.35, and so on to 2.

Beginning again at 1, and calling the 1 ten, then between 10 and 20 each of the principal subdivisions will be equal in value to one unit, and the *reading* will be 10, 11, 12, 13, 14, and so on to 20. And if between the long marks there be two spaces, each of these must be reckoned one-half, or .5; but if there be five spaces between the principal minor subdivisions, then each of these must be reckoned one-fifth, or .2. Suppose there are fifty spaces or divisions on the rule between 1 and 2, or between 10 and 20, calling the 1 ten and the 2 twenty, then, between 10 and 20, if you reckon the value of each division, you will *read* 10.2, and 10.4, and 10.6, and 10.8, and $10\frac{10}{10}$, or 11, and 11.2, and 11.4, and 11.6, and 11.8, and so on to 20. Between 20 and 30, if there be ten spaces or divisions, then each division must be reckoned one, but if there be twenty spaces between 20 and 30, each division will be one-half, or .5; and the computation or *reading off* will be the same between 30 and 40, 40 and 50, 50 and 60, and so on to 80. Between 80 and 90, and 90 and 100, if there be but five divisions, then the value of each will be two. Between 100 and 200, (calling the 1 at the middle of the line 100, and the 2 following 200,) as there are ten long marks, each of these will count ten; and if between these there are but two divisions, each division will count two; but if, instead

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of two spaces between the long marks, there be five, (which is the more usual number,) then each of these spaces must be called 2; in which case you will *read off* 102, 104, 106, 108, and so on to 200. Between 200 and 300, if there be but ten spaces, each space will count ten; but if there be twenty divisions each must be reckoned five; and in this case half of one of the divisions would be the half of five, or 2.5. Between 300 and 400, as there are but ten divisions laid down on the rule, each will count ten; and the half of one of the divisions must be reckoned five; and a fourth of one of the spaces would be 2.5; and from 400 to 1000 the computation will be the same, except on those rules which have but five spaces between 800 and 900, and 900 and 1000, when each space must be counted twenty; and the half of one of the divisions must be called ten.

Beginning again at the commencement of the scale, and calling the 1 at the beginning one thousand, then the 2 following will be two thousand; and each of the long marks between 1000 and 2000 must be called one hundred: and if there be but two spaces between the long marks, then each must be called fifty; but if there be five, then each mark or division will count twenty; and the computation, or *reading off*, will be the same between 3000 and 4000, and so on.

The line of numbers, called the D line, contains the *square* and *cube roots* of the numbers expressed on the *A*, *B*, or *C* line; but the manner of *reading off* is the same on this line as on the others.

Under the girt, or D line, is a scale of equal parts, which is useful for many purposes, but more particularly so in drawing plans and mathematical figures, and for laying out work. The lowest line is a scale of inches, one inch being divided into twelve equal parts. The line next above is a scale of equal parts three-fourths of an inch apart; and one of these equal parts is divided into twelve equal parts, one of these minor divisions being of course one-sixteenth of an inch. The line next above is a scale of half-inches, one of the parts being divided into twelve equal parts, and of course expressing one-

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twenty-fourth of an inch; and the upper, or top line, is a scale of fourths of an inch.

The lines A and B are used in multiplying, dividing, and stating proportions; and the lines C and D are used in gauging, casting, or finding the contents of squares and solids, and in extracting the square and cube roots.

BELCHER'S ENGINEER'S SLIDING RULE.

This rule is generally preferred to the carpenter's rule by engineers and machinists. The first three lines (viz. the A, the B, and the C lines) are in all respects the same as the corresponding lines on the carpenter's rule, represented in the cut; but the D line, instead of commencing with 4 on the left end of the scale, commences with 1, and ends with 10 on the right. The principle on which this line is constructed is, nevertheless, the same as that of the corresponding line on the carpenter's rule; and all the *gauge points* for the latter rule may be used, and with the same facility, by those who possess the former rule; though there are certain methods of solving problems by the engineer's rule, which are not applicable to the carpenter's rule.

The D line, as laid down on the engineer's rule, is more simple and easy to be understood than the same line on the carpenter's rule; as it differs in no respect from the other three lines, except that the spaces between the several divisions are twice as great. And since 1 on C stands over 1 on D, (when the slider is in its usual position, or is *shut*,) the numbers on C are the squares of those standing under them on D; and the numbers on D are the square roots of those directly over them on C. Hence the area of any square, or the side of any square, (containing a given area,) may be found by means of this rule without moving the slider.

The principal *gauge points*, on the line D, will be found between 1 and 2, or between 10 and 20, on the *left end* of the scale;—

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whilst the same *gauge points* on the carpenter's rule must be sought for on the *right* of the 1, near the centre of the rule.

Under the line D there is a *table* of *gauge points* for square prisms, cylinders, and globes, and likewise *gauge points* for pumping engines, for the weight of bodies, for regular polygons, for circles, squares, trigons, &c. These *gauge points* are expressly prepared for this rule, and some of them are not applicable to the carpenter's rule; though the same *gauge points may be used on both rules in all cases*, when the solution is effected by employing two lines only; *and there is no necessity of ever employing more than two lines, at the same time, in solving a problem by either of these rules.*

To illustrate the method of solving problems by employing three or four lines, and to show the difference between the two rules, we will introduce, in this place, a few examples, which more properly belong to another part of this work.

EXAMPLES.

1. Suppose a piece of timber is 9.8 inches square and 20 feet long; how many solid feet does it contain?

Ans. 13.3 cubic feet.

Set 20, the length, found on B, under 144 on A, and over 9.8, found on D, will be found 13.3 feet on the line C.

2. A cylinder is 30 feet long and 12.73 inches in diameter; how many cubic feet does it contain? *Ans.* 26.5 feet.

Set 30, the length, found on B, under 183.3, the *gauge point* on A, and over 12.73, found on D, will be found 26.5 feet on C.

3. A cylinder is 6 inches in diameter and 6 inches in length; how many cubic inches does it contain?

Ans. 169 cubic inches.

Set 6 on B under 1.273 on A, and over 6 on D, will be found 169 on C.

4. A cylinder is 6½ feet long and 20 inches in diameter; required its cubic content. *Ans.* 14.2 cubic feet.

Set 6½ on B under 183.3 on A, and over 20 on D, are 14.2 feet on C.

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5. What is the content of a cylinder, the content of a globe, and the content of a cone, the diameter of each being 12 inches, and the altitude of the cylinder and cone each 12 inches?

Ans. 1356; 904; and 452 cubic inches.

Set 12 on B under 1.273 on A, and over 12 on D, are 1356 cubic inches, the content of the cylinder. Set 12 on B under 1.91 on A, and over 12 on D, are 904 inches, the content of the globe. Set 4 on B, (viz. one-third of the altitude of the cone,) under 1.273 on A, and over 12 (the diameter of the base of the cone) on D, are 452 inches on C.

These problems may be solved with equal or greater facility by the carpenter's rule, but they cannot be solved in this manner, in consequence of the line D on said rule not being adapted to this method.

The *gauge points* for wine, ale, and imperial gallons, (by the above method,) in a cylinder are 294, 359, and 353 on the line A; and 231, 282, and 277.28 for vessels having square bases; and 2150.4 and 2218 for bushels and imperial bushels for square vessels; and 2738.3 and 2824 for cylindric vessels.

The weight of bodies may be found in the same manner by means of the gauge points laid down on the face of this rule.

A TABLE OF GAUGE POINTS.

To find the solid content of shafts or prisms, that are polygonal-sided, by this method.

No. of Sides.	Gauge Points.
3	2.309
5	.581
6	.385
7	.275
8	.207
9	.161
10	.130
11	.107
12	.089

RULE.

As the length of the prism on B is to the gauge point on A, so is the length of one of its sides on D to the solid content in inches on C; or set the length upon B to the gauge point upon A, then against the length of one of its sides on D, you have the content in cubic inches on C.

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What is the solid content (in cubic inches) of a triangular prism, whose height is 24 inches, and the length of each side 12 inches? *Ans.* 1496 inches.

¶ 2. MULTIPLICATION ON THE SLIDING RULE.

Find the multiplier on the B line, and place it under 1 *on the A line; then find the multiplicand on the A line, and under it will be found the product or answer on the line B.*

EXAMPLES FOR PRACTICE.

1. To multiply by 3: bring 3 on the line B under 1 on the line A, then under 2 on A you will find 6 on the line B, which is the product of 3 times 2; and under 3 you will find the product of 3 times 3, or 9; and under 4 you will find 12, and under 5, 15; under 6, 18; under 10, 30; under 12, 36; under 20, 60; under 25, 75; under 30, 90; under 35, 105; under 40, 120; under 50, 150; under 60, 180; under 75, 225; under 80, 240.

2. To multiply by 5: move the slider till you bring 5 on the line B under 1 on the line A, then under 12 on the A line you will find the product, 60, on the line B; and under 20 you will find 100; under 25, 125; or calling the multiplier 5 fifty, then under the 12 you will find 600; under 20, 1,000; under 25, 1,250; under 30, 1,500; under 45, 2,250; and under 75, 3,750.

3. To multiply by 2.5: bring 2.5 under 1, and under 2 you will find the product of 2 times 2.5, viz. 5; under 4 you will find 10; under 5, 12.5; under 6, 15; under 12, 30; under 20, 50; and under 40, 100. Or, call the 2.5 twenty-five, and under 2 you will have 50; under 3, 75; under 4, 100; under 5, 125; under 10, 250; under 20, 500; under 25, 625; under 35, 875; under 40, 1,000; under 45, 1,125; under 50, 1,250; under 55, 1,375; under 60, 1,500; under 80, 2,000;

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under 85, 2,125; under 90, 2,250; under 100, 2,500: under 120, 3,000; and under 125, 3,125.

4. Multiply 7.3 by 20.2, and the product will be 147.5; or multiply 5.7 by 13.5, and the product will be 76.95; or multiply 9.4 by 7.6, and the product will be 71.4; and multiply 6.8 by 13.1, and the product will be 89.1; and multiply 18.6 by 6.2, and the product will be 115.3; and multiply 2.7 by 6.8, and the product will be 18.4. Or, calling the 2.7 two hundred and seventy, under 2 you will find 540; under 6, 1,620; under 6.8, 1,896; under 7.5, 2,025; under 8, 2,160; and under 8.6, 2,322.

With a good rule, multiplication may be performed with perfect accuracy, when the product does not exceed 5,000, by determining the unit figure in the mind.

¶ 8. TO DIVIDE BY THE SLIDING RULE.

Find the DIVISOR *on the line B, and place it under* 1 *on the line A; then find the* DIVIDEND *on the line B, and over it you will find the* QUOTIENT *on the line A.*

EXAMPLES.

1. To divide 48 by 12, find the divisor, 12, on the line B, and place it under 1 on the line A; then find the dividend on B, and over it you will find the quotient, 4, on A; and over 60 you will find the quotient of 60 divided by 12, viz. 5; and over 84 you will find 7, the quotient of 84, divided by 12; and over 90, 7.5; or, calling the 9 nine, the quotient will be .75; or, calling the 9 nine hundred, the quotient will be 75; or, calling the 9 nine thousand, the quotient will be 750.

2. Divide 48, and 60, and 72, by 24. Bring the divisor, 24, on B, under 1 on A, then over 48 on B you will find 2, the quotient, on A; and over 60 you will find 2.5; over 72, 3; over 70, 2.91; over 95, 3.95; over 100, 4.15, &c.

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3. Divide the following numbers, 66, 77, 88, 110, 150, 220, 300, 440, 600, 700, 1,000, 1,500, 2,200, and 3,000, by 44. Answers, or quotients, 1.5; 1.75; 2; 2.5; 3.3; 5; 6.8; 10; 13.6; 15.9; 22.72; 34; 50; 68.

4. Divide 120, 150, 180, 240, 300, 450, 600, 700, 800, 1,200, 3,000, 4,400, 5,000, and 6,000, by 60. The quotients are respectively, 2; 2.5; 3; 4; 5; 7.5; 10; 11.66; 13.33; 20; 50; 73.33; 83.33; 100.

5. Divide 6.4, 4, 2, 1.2, 1, and .8, by 1.6. Quotients in order, 4; 2.5; 1.25; .75; .625; and .5.

¶ 4. THE RULE OF THREE ON THE SLIDING RULE.

Find the FIRST *term on the A line, and the* SECOND *term on the line B: then place the* SECOND *term under the* FIRST, *and find the* THIRD *term on the line A, and under it you will find the* FOURTH *term, or answer, on the line B.*

EXAMPLES.

1. If 3 pounds of beef cost 21 cents, what will 2 pounds cost? what 6 lbs.? 8 lbs.? 11 lbs.? 15 lbs.? 22 lbs.? 27 lbs.? 45 lbs.? 60 lbs.? and 100 lbs.?

Find 21, the second term, on the line B, and place it under 3, the first term, on A; then find 2 on A, and under 2 you will find the price, or cost of 2 pounds, on B, viz. 14 cents; and under 6 you will find the cost of 6 lbs., viz. 42 cents; under 8 you will find the cost of 8 lbs., viz. 56 cents; under 11, 77; under 15, 1.05, or $1.05; under 22, 1.54; under 27, 1.89; and under 45, 3.15, or $3.15.

2. A school district wishes to raise a tax of $30, to be made up on the scholars, each patron of the school being required to pay such proportion of the $30, as the number of days he has sent to the school bears to the number of days all the pupils

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in the school have attended. If, then, the whole number of days be 3,600, how much must A pay for 240 days' tuition? how much must B pay for 200? C for 150? D for 120? E for 100? F for 90? G for 70? H for 60? I for 36? J for 30? K for 20? L for 15? M for 8? N for 4? and O for 1? In cases like the present, it will be found most convenient to place the first term of the proportion on the line A, to the right of the middle of the rule. Hence, in this case, draw the slider a little to the right, placing 3, or 30, on B, under 3.6, or 3,600, on A; then, since under the whole number of days on the A line, you have the whole amount of money to be raised, so under any number of days found on the A line, will be found the tax, or tuition to be paid for that number of days, on the line B. Thus, under 240 on the A line, we find 2, or 2 dollars, on the line B, which is the amount of A's tax; under 200 days we find $1.66 for B's tax; under 150, $1.25; under 120, 1 dollar; under 100, 83 cents; under 90, 75 cents; under 70, 58 cents; under 60, 50 cents; under 36, 30 cents; under 30, 25 cents; under 20, 16⅔ cents; under 15, 12½ cents; under 8, 6 cents and 7 mills, or .067; under 4, .0335; under 1, .0084, or 8 mills and 4 tenths nearly.

3. A school tax of $45 is to be raised; the whole number of days is 4,000; what must C pay for 400 days' tuition? D for 200? G for 80? H for 35? and K for 30 days?

Answers. C must pay $4.50; D, $2.25; G, 90 cents; H, 39½ cents; and K, 34 cents.

4. What is the interest on 100 dollars at 9 per cent. for 150 days, for 100 days, for 80 days, for 70 days, for 50 days, for 30 days, for 12 days, and for 3 days?

In calculating interest for days, it is customary to call 30 days one month, and 360 days one year; consequently make 360 days the first term, and 9 dollars the second, and proceed as in the last example. Then, under 150 days you will find $3.75; under 100, $2.50; under 80, 2 dollars; under 70, $1.75; under 50, $1.25; under 30, 75 cents; under 12, 30 cents; and under 3 days, 7½ cents.

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5. If the grand list of a certain town be $250,000, and the tax to be raised on the property of that town is $2,100, what will be the tax on an estate valued at $2,500? and what on one valued at $1,000, on one valued at $650, on one valued at $500, on one valued at $400, on one valued at $250, on one valued at $200, on one valued at $100, on one valued at $75, on one valued at $25, and on one valued at 8 dollars? And what will be the tax on one dollar?

In this example $250,000 is the first term, and 2,100 the second; and having placed the first term over the second, (as in the last example,) to avoid any mistake in *reading off*, let the learner run along the line A from $250,000 till he comes to the end of the rule on the left hand, which brings him to 10,000; then beginning at A on the right, and calling the ten on the rule 10,000, let him continue the computation from A towards the left till he comes to the starting-point, when the 250,000 will count 2,500; then let him commence with the second term, 2,100, on B, and proceed in the same manner till he has made the circuit of the line B, when under the first term he will find 21, or $21, which would consequently be the tax on $2,500; and under 1,000 we find 8.4, or $8.40, for the tax on $1,000; and under 650, $5.46; under 500, $4.20; under 400, $3.36; under 250, $2.10; under 200, $1.68; under 100, 84 cents; under 75, 63 cents; under 25, 21 cents; under 8, $6\frac{3}{4}$ cents; and under 1, $8\frac{4}{10}$ mills.

VULGAR AND DECIMAL FRACTIONS.

To reduce a vulgar fraction to its equivalent decimal expression, the proportion is, as the denominator upon A is to 1 on B, so is the numerator upon A to the decimal required on B.

I.

Reduce $\frac{1}{4}$ to its decimal expression.

Set 1 upon B to 4 upon A, then against 1 upon A is .25, the answer, upon B.

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II.

Reduce $\frac{9}{12}$ to a decimal.

Set 1 upon B to 12 upon A, and against 9 on A is .75, the answer, on B.

III.

What is the decimal of $\frac{18}{28}$?

Set 1 on B to 28 on A, and against 18 on A stands .643 on B, the decimal required.

To find a multiplier to a divisor that shall perform the same by multiplication as the divisor would do by division; the proportion is, as the divisor upon A is to unity or 1 upon B, so is unity on A to the multiplier required on B.

I.

Suppose 25 to be the divisor, what will be the multiplier to that number?

Set 1 upon B to 25 upon A, and against 1 on A is .04, the multiplier on B.

II.

What will be the multiplier to 80?

Set 1 on B to 80 on A, and against 1 on A is .0125 on B.

III.

What will be the multiplier to 40?

Set 1 on B to 40 on A, and against 1 upon A is .025, the answer, upon B.

Having a multiplier given to find a divisor.—The proportion is, as the multiplier upon A is to 1 upon B, so is the divisor upon B to 1 upon A.

I.

Let .04 be the multiplier given to find a divisor.

Set 1 upon B to .04 upon A, and against 1 upon A is 25 upon B.

II.

What will be the divisor for .0125? *Ans.* 80.

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Set 1 upon B to .0125 upon A, and against 1 on A is 80 on B.

III.

What will be the divisor for .7854? *Ans.* 1.273.

Set 1 upon B to .7854 upon A, and against 1 on A is 1.273, the answer, on B.

¶ 5. EXTRACTION OF THE SQUARE ROOT.

The square root of any number is that number which, multiplied by itself, or *squared*, will produce the given number. Thus—

The square root of	1	4	9	16	25	36	49
is	1	2	3	4	5	6	7

To find the square root of any number:—

Divide the given number into periods of two figures each, by putting a dot over the unit figure, and every second figure from the place of units, and the number of dots will be equal to the number of figures in the required root.

Then seek the greatest square number in the left-hand period, and place it in the quotient, for the first figure of the root, and place its square under the left-hand period, and subtract it from that period, and to the remainder annex the figures in the following period.

Double the root, and place it on the left hand for a divisor, and seek how often the divisor is contained in the dividend, (always excepting the right-hand figure of the dividend,) *and place the figure in the quotient, and also annex it to the divisor; then multiply the divisor with the annexed figure of the quotient by that figure, and place the product under the dividend, which subtract from the dividend, and to the remainder annex the following period, and proceed as before, and so on till all the periods are exhausted.*

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EXAMPLES.

1. What is the square root of 119025?

```
. . .
119025(345 Ans.
 9     /
------
64)290
   256
------
685)3425
    3425
   ====
```

2. What is the square root of 436.5?

```
. . . . . .
436.50000000(20.8925+Root.
4
------
408)3650
    3264
------
4169)38600
     37521
------
41782)107900
      83564
------
417845)2433600
       2089225
------
        344375
       ======
```

3. What is the square root of 80? *Ans.* 8.94427+.

4. What is the square root of 22071204? *Ans.* 4698.

5. What is the square root of 2.7109? *Ans.* 1.6462+.

To extract the square root of a vulgar fraction, extract the root of the numerator and denominator; or reduce it to a decimal, and then extract its root.

6. What is the square root of $\frac{9}{36}$? *Ans.* $\frac{3}{6}$.

7. What is the square root of $\frac{215}{441}$? *Ans.* 0.89802+.

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To find a *mean proportional* between two numbers:—

Extract the square root of the product of the given numbers.

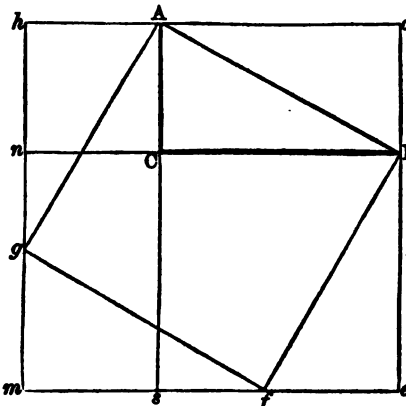
1. What is the geometrical mean proportional between 3 and 12? *Ans.* 6.

2. What is the *mean proportional*, or, strictly, the geometrical mean proportional between 20 and 30? *Ans.* 24.49.

¶ 6. RIGHT-ANGLED TRIANGLE.

In any right-angled triangle the square of the hypotenuse is equal to the sum of the squares of the base and perpendicular.

To demonstrate this useful proposition,—Let ABC be a right-angled triangle. On the base, BC, draw the square B*es*C; and



on the perpendicular, AC, draw the square AC*nh*; and on the hypotenuse, AB, draw the square A*gf*B; then from A, draw A*d* parallel to CB, and equal to the same, and join *d*B; then from *n*, draw *nm* parallel to C*s*, and equal to C*s*, and join *ms*.

Now since the four right-angled triangles, AB*d*, B*ef*, *fmg*, and *gh*A are equal, their sides and angles being equal; and since the rectangle A*d*BC is equal to two of these triangles;

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and the rectangle C*smn* is equal to the rectangle A*d*BC, its sides being of the same length, it is manifest that the sum of these two rectangles is equal to the sum of the four triangles, A*d*B, B*ef*, *fgm*, and A*gh*. Now if we subtract the sum of the two rectangles, A*d*BC and C*smn*, from the square *hdem*, there will remain the squares drawn on the base and perpendicular of the triangle ABC, viz., the squares AC*nh* and CB*es*; and if we subtract from the same square the four triangles, AB*d*, A*gh*, *gfm*, and B*fe*, there will remain the square AB*fg* on the hypotenuse. But these four triangles are equal to the sum of the two rectangles, A*d*BC and C*smn*: since, therefore, we have subtracted equal quantities, in both cases, from the square *hdem*, it follows that the remainders will be equal; and consequently, the square on the hypotenuse is equal to the sum of the squares on the base and perpendicular.

The base and perpendicular of a right-angled triangle being given, to find the hypotenuse:—

Extract the square root of the sum of the squares of the base and perpendicular.

EXAMPLES.

1. The wall of a town is 25 feet in height, and being surrounded by a moat 30 feet in breadth, I desire to know the length of a ladder which will reach from the outside of the moat to the top of the wall. *Ans.* 39.05+ feet.

2. One side of a rectangle being 45 rods, and the other side 60 rods, I require the length of the diagonal, or the distance between the opposite corners. *Ans.* 75 rods.

3. If a room is 9 feet in height, and 25 feet long, and 15 broad, what will be the length of a diagonal between the opposite corners? *Ans.* 30.512+ feet.

To find the diagonal in the above example, add together the squares of the length, breadth, and height of the room, and extract the square root of the sum.

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The hypotenuse and base or perpendicular being given, to find the other leg of the triangle:—

From the square of the hypotenuse subtract the square of the given leg, and extract the square root of the remainder.

EXAMPLES.

1. If the diagonal of a rectangle be 50 rods, and its breadth 30, what is its length? *Ans.* 40 rods.

2. If the foot of a ladder 75 feet in length be 60 feet from the base of a wall, and the other end rests on the top of the wall, what is the height of the wall? *Ans.* 45 feet.

¶ 7. EXTRACTION OF THE CUBE ROOT.

To extract *the cube root* is to find out a number, which being raised to the third power, (or, which being multiplied into itself, and then into that product,) will produce the given number.

To extract the cube root:—

1. *Divide the given number into periods of three figures each, by placing a dot over the unit figure and every third figure from the place of units; then find the greatest cube in the left-hand period, and subtract it therefrom, placing the root in the quotient; then to the remainder annex the figures of the following period for a dividend.*

2. *Square the quotient, and triple the square for a divisor. Find how often it is contained in the dividend, (excepting units and tens,) and put the number in the quotient.*

3. *Square the last figure in the quotient, and annex its square to the divisor.**

* *When the quotient figure is* 1, *or* 2, *or* 3, *its square being less than* 10, *a cipher must be put in the place of tens.*

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4. Triple the last figure in the quotient, which product multiply by the former figure or figures in the root, and putting it under the divisor, units under tens, and tens under hundreds, add them together, and multiply the sum by the last figure in the quotient, and, placing the product under the dividend, subtract it therefrom, and to the remainder annex the figures in the period following, and proceed as before; and so continue to do till all the periods are exhausted.

EXAMPLES.

1. What is the cube root of 673373.097125?

Square of $8\times3=192$, divisor. 673373.097125(87.65

512

Square of 7 annexed to $192=19249$ 161373

$7\times3\times8=168$ 146503

20929 14870097

7 13718376

146503 1151721125

1151721125

Square of $87\times3=22707$, divisor.

Square of 6 annexed = 2270736

$6\times3\times87=1566$

2286396

6

13718376

Square of $876\times3=2302128$.

Square of 5 annexed = 230212825

$5\times3\times876=13140$

230344225

5

1151721125

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2. What is the cube root of 99252.847?

Square of $4\times3=48$, the divisor. 99252.847(46.3 *Ans.*
64

Square of 6 annexed to 48=4836 35252
$6\times3\times4=$ 72 33336

5556 1916847
6 1916847

33336

Square of $46\times3=6348$ divisor.

Square of 3 annexed to 6348=634809*
$3\times3\times46=$ 414

638949
3

1916847

3. What is the cube root of 32461759? *Ans.* 319.

4. What is the cube root of 33.230979637?
Ans. 3.215+.

5. What is the cube root of $9\frac{1}{8}$? *Ans.* 2.092+.

6. A cubic block of marble contains 389017 solid feet,—I demand the superficial contents of its 6 sides.
Ans. 31914.

To find two mean proportionals between two given numbers:—

Divide the greater number by the less, and the cube root of the quotient multiplied by the less extreme, or smaller number, will give the less mean proportional, which being multiplied by the said cube root will give the greater.

EXAMPLES.

1. What are the two mean proportionals between 6 and 162? *Ans.* 18 and 54.

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2. What are the two mean proportionals between 4 and 108? *Ans.* 12 and 36.

3. There are 4 weights which will weigh any number of pounds from 1 to 40, the least being 1 pound and the greatest 27, the other two are required. *Ans.* 3 and 9.

To find the side of a cube that shall be equal in solidity to any given solid, as a globe, cylinder, prism, cone, &c.:—

Extract the cube root of the solid contents of the given body.

EXAMPLES.

1. If the solid contents of a globe be 10648, what is the side of a cube of equal solidity? *Ans.* 22.

2. There is a cubical vessel, whose side is 12 inches; it is required to find the side of another vessel that will contain three times as much. *Ans.* 17.307 inches.

¶ 8. BIQUADRATE ROOT.

The biquadrate root (or fourth root) is that number which being involved four times into itself, will produce the given number.

To find the biquadrate root:—

Extract the square root of the given number, and then extract the square root of that root.

EXAMPLES.

1. What is the biquadrate root of 5719140625?

Ans. 275.

2. If the principal and interest on one dollar for four years (the interest being compounded yearly) be 1.21550625, what is the rate per cent.? *Ans.* 5 per cent.

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To find the sixth root of any number:—

Extract the square root of the cube root of the given number; and, to find the eighth root, extract the square root of the square root of the square root of the number; to find the ninth root of any number, extract the cube root of the cube root of the number; to find the fifth root of any number—if the number be less than 100, from half the sum of the biquadrate and sixth roots, subtract $\frac{10}{71}$ of the difference of said roots; but if the number be greater than 100 and less than 500, from the said half sum subtract $\frac{1}{7}$ of said difference, but when the number is more than 500, subtract $\frac{1}{6}$ of said difference, and the remainder will be the fifth root nearly.

EXAMPLES.

1. If the amount of one dollar for six years at compound interest be $1.418519112256, what is the rate per cent.?

Ans. 6 per cent.

2. What is the eighth root of 5236? *Ans.* 2.91659.

3. What is the sixth root of 894? *Ans.* 3.10377.

4. What is the fifth root of 894? *Ans.* 3.8919 nearly.

5. What is the fifth root of 49? *Ans.* 2.17754.

6. What is the ninth root of 5236? *Ans.* 2.00499.

7. If the amount of one dollar for nine years is $1.55132, what is the rate per cent., compound interest?

Ans. 5 per cent.

¶ 9. SQUARE ROOT ON THE SLIDING RULE.

To *square* any number by the sliding rule; or, to extract the *square root* of any number:—

Place 1 *on the line C over* 1 *or* 10 *on the line D, and over any number found on the line D, will be found its square on C; or, under any number found on the line C, will be found the square root on D.*

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Having *set* the slider as directed, call the 10 on the line D one, and over it you will find its square, viz. 1; or, if you call the 10 on D 10, then the 1 over it must be called 100; or, if you call the 10 on D 100, then the 1 over it will count 1,000, &c.; and over any number on D you will find its square on C, calling the 10 at the middle of the line D, 1, or 100, or 1,000, or 10,000, and counting towards the right, if the number be between 1 and 4, or 10 and 40, or 100 and 400, or 1,000 and 4,000, &c., but if the number, whose square is required, be between 4 and 10, or between 40 and 100, or between 400 and 1,000, or 4,000 and 10,000, find the given number on the line D to the left of the 10 at the middle of the scale. Or you may commence with the 4 at the left end of the D line, and calling it $\frac{4}{10}$, and counting towards the right, the 10 at the middle of the line will count 1, and the 40 at the end of the line will count 4; or, calling the 4 at the left end of the line D, 4, and counting towards the right, you will find 10 at the middle of the line, and the 40 at the end of the line will count 400; you may then commence again with the 4 at the left end of the D line, and calling it 400, at the middle of the rule you will find 1,000, and at the end 4,000, &c. The square root of any number will be found on D, directly under the number on the C line; but in *reading off*, to avoid mistake, you must commence with 1 on C, at the middle of the rule, and read towards the right or left as directed above, when *reading off* the squares of numbers.

When the slider has been *set*, as directed above, there is a part of the line D on the right, which does not come under the slider, which renders it impossible to *read off* the square of a number between 3.15 and 4, or between 315 and 400, without *resetting* the slider, so as to bring this portion of the rule, or scale, under the line of numbers on C. In this, and all similar cases, *notice* what figure or portion of the slider stands over 4 on the line D at the *left end* of the line; then set the same figure, or part of the line C, over the 4 or 40 on D, at the right hand, and the difficulty will be overcome.

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EXAMPLES.

1. What is the square of 1? what of 9 tenths? 7 tenths? 5 tenths? 35 hundredths? and of 2 tenths?

Having *set* the slider, as directed in the rule, over 10 on D, calling the 10 one, you will find 1, its square, and running to the left on D, over 9 tenths, you will find its square, 81 hundredths; over 7 tenths, you will find 49 hundredths; over 5 tenths, 25 hundredths; and (sliding the slider to the right, placing 16, or 16 hundredths, on C, over the 40, or 4 tenths, on D, at the right end of the line) over 35 hundredths we have 1225 ten-thousandths; and over 2 tenths you will find 4 hundredths.

2. What is the square of 1.5? of 2? of 3? of 3.5? of 4? of 6? of 10? of 12? of 15? of 25? of 30? of 40? of 60? of 75?

Having *set* the slider, as directed in the rule, over 1.5 you will find 2.25; over 2, 4; over 3, 9; over 3.5, 12.25; over 4, 16; over 6, 36; over 10, 100; over 12, 144; over 15, 225; over 25, 625; over 30, 900; over 40, 1600; over 60, 3,600; and over 75, 5,625.

3. What is the square of 25 hundredths? of 49 hundredths? of 144? and of 33.5?

Ans. .0625; .24; 20,736; 1125 nearly.

4. What is the *square root* of 1? of 81? of 49? of 25? of 2? of 3? of 4? of 10? of 20? of 30? of 40? of 60? of 88? of 144? of 225? of 400? of 625? of 1,000? of 1,400? of 2,025? of 2,500? of 3,600? of 6,400? of 8,100? of 40,000? of 90,000?

Having *set* the slider, as directed in the rule, under 1 we find its root, calling the 10 on D, 1; under 81, we find its root, viz. 9; under 49, 7; under 25, 5; under 2, 1.414; under 3, 1.732; under 4, 2; under 10, 3.16; under 20, 4.47; under 30, 5.47; under 40, 6.32; under 60, 7.75; under 81, 9; under 88, 9.41; under 144, 12; under 225, 15; under 400, 20; under 625, 25; under 1,000, 31.62; under 1,400, (having *reset* the slider,) 37.4; under 2,025, 45; under 2,500, 50; under 3,600, 60; under 8,100, 90; under 40,000, 200; and under 90,000, we find 300 for the root, &c.

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¶ 10. ON THE CONSTRUCTION OF THE LINES A, B, C, AND D.

As before stated, in the general description of the Sliding Rule, the divisions on the lines A, B, and C are exactly alike, the length of the spaces being to each other as the logarithms of the numbers which they represent. If the natural numbers be considered as terms in an infinite series of proportionals, beginning at unity, and either increasing or decreasing to infinity, the logarithm of any number is its distance from unity in that series; and the logarithms of the natural numbers are so related to each other, and to the numbers which they represent, that the sum of any two logarithms is the logarithm of the *product* of the two numbers for which they stand; and the difference of any two logarithms is the logarithm of the *quotient* of one of the numbers divided by the other; and twice the logarithm of any number is the logarithm of the square of that number.

Hence the spaces, or the divisions, on the lines A, B, and C, are to each other as the logarithms of the natural numbers which they represent, the distance from 1 to 3 being as much greater than the distance from 1 to 2, as the logarithm of 3 is greater than the logarithm of 2; and the distance from 1 to 4 is as much greater than the distance from 1 to 3, as the logarithm of 4 is greater than the logarithm of 3; and so on through the scale. Four being the square of 2, its logarithm will be double the logarithm of 2; and consequently, the distance from 1 to 4 will be twice the distance from 1 to 2; and because the square of 3 is 9, the distance from 1 to 9 will be twice the distance from 1 to 3; and the distance from 1 to 16 will be, for the same reason, twice the distance from 1 to 4, &c.

Since, therefore, the numbers on the lines A and B are so laid down on the scale, that their distances from unity are as their logarithms compared with unity, it follows that if we bring one of the factors of any number under unity, the other factor will stand over the product; and hence we can multiply or divide any number by bringing the multiplier or divisor under

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unity, and seeking for the product under the multiplicand, and for the quotient over the dividend, as has been demonstrated. See ¶ 2 and 3.

And since twice the logarithm of any number is the logarithm of the square of that number, and since the numbers on the line C are the squares of those on the line D, it follows that the distances between the numbers on the line D must be twice as great as the distances between the corresponding numbers on the line C. Thus, the distance from 1 to 2 on D, is equal to the distance from 1 to 4 on C; and the distance from 1 to 4 on D, is equal to the distance from 1 to 16 on C, and so on. Hence it follows, that if we place 1 on C over 1 on D, under any number found on C, we shall find its square root on D; or over any number found on D we shall find its square on the line C, as we have shown in the extraction of the square root, and in the squaring of numbers. (See the description of the engineer's rule, ¶ 1.)

¶ 11. CUBE ROOT ON THE SLIDING RULE.

To cube any number by the sliding rule:—

Place the given number on the line C over 1 *on the line D, and over the given number found on the line D will be found its cube, or third power, on the line C.* Or, place the square of any number found on the line C, over 1 on the line D, and over the given number found on the line D, will be found its fourth power on the line C.

To extract the CUBE ROOT *by the sliding rule:—*

Move the slider either way, until the number, whose root is required, stands *over* the same number on the line D, that 1 on D stands *under* on C; and then will the number that stands over unity on D, be the root required.

Or, *Draw out the slider and reverse its ends, so as to bring the line B, inverted, over the line D; then find the given num-*

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ber on the line B, and place it over 1 *on D; then look along the scales, or lines B and D, until you find the numbers or divisions on each coinciding, or the same number on B standing over the same on D; then will that number be the required root.*

The slider being reversed, the numbers on the line B will increase from right to left, whilst those on D will increase from left to right; and consequently, the numbers on B must be counted from right to left, and those on D from left to right. It will also occasionally happen, that the numbers on the lines B and D will agree, or coincide, in two different places, only one of which will give the true root; hence some degree of caution is required in *reading off*.

EXAMPLES.

1. What is the cube of 3? of 12? of 15? of 18? of 5? of 20? of 16? of 25? and of 30?

Answers. 27; 1,728; 3,375; 5,832; 125; 8,000; 4,096; 15,625; and 27,000.

2. What is the cube root of 8?

Having reversed the slider, and *set* 8 on B over 10 on D, look along the scale towards the right, and you will find 2 on B exactly over 2 on D; 2 therefore is the cube root of 8, as you may prove by raising it to the third power.

3. What is the cube root of 27?

Having placed 27 on B over 1 on D, look along the scale towards the right, and you will find 3 on D directly under 3 on B; 3 therefore is the cube root of 27.

4. What is the cube root of 125?

Having set the slider as directed, look along the scale towards the left, and you will find 5 on B over 5 on D; 5 therefore is the cube root of 125.

5. What is the cube root of 216, or the side of a cubic block which contains 216 solid feet?

Having *set* the slider, to the left of the middle of the rule you will find the root 6, on B, over 6 on D.

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6. What is the cube root of 729? of 1,728? of 3,375? of 8,000? of 10,000? of 12,000? of 15,625? of 27,000? of 30,000? of 45,000? of 60,000?

Answers. 9; 12; 15; 20; 21.52; 22.89; 25; 30; 31.06; 35.6; 39.16.

7. There are 2,150 cubic inches in a bushel; required the side of a cube that will contain 1 bushel.

Ans. 12.88 inches nearly.

8. There are 282 cubic inches in an ale gallon; what is the side of a cube that will hold an ale gallon?

Ans. 6.57 inches.

¶ 12. TABLES.

1. *Long Measure.*

3	barleycorns make	1 inch.
12	inches	1 foot.
3	feet	1 yard.
5½	yards, or 16½ feet	1 rod or pole.
40	rods, or 220 yards	1 furlong.
8	furlongs, or 320 rods	1 mile.

The surveyor's chain is composed of 100 links, and

25	links make	1 rod.
4	rods, or 100 links	1 chain.
80	chains, or 5,280 feet	1 mile.
6076	feet	1 *nautical* mile.
3	miles	1 league.
60	geographical, or 69⅛ statute miles	1 degree.

The mean length of a degree on the earth's surface is 69.01745 miles.

A French mile is	6,392 feet.
A Scotch mile is	7,332 "
An Irish mile is	6,724 "
A Russian mile is	3,300 "

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A Spanish mile is - - -	15,084 feet.
A German mile is - - -	17,608 "
A Roman mile is - - -	4,909 "
A French league is - - -	8,799 "
A French great league is - -	13,200 "
An Italian mile is - - -	4,401 "
A Polish mile is - - -	13,200 "
A Swedish or Danish mile is -	21,699 "
A Hungarian mile is - - -	25,400 "
A Dutch mile is - - -	24,303 "
An Arabian mile is - - -	6,444 "
A Turkish Beri is - - -	5,478 "
107 French feet make - -	114 English ft.
A French metre is - - -	39⅓ inches.
A French kilometre is - -	3,279 feet.

2. *Land or Square Measure.*

144 square inches make - -	1 foot.
9 square feet - - - -	1 yard.
30¼ square yards, or 272¼ square ft.	1 rod.
40 square rods - - - -	1 rood.
4 roods, or 160 square rods -	1 acre.
640 acres - - - - -	1 square mile.
625 square links - - - -	1 square rod.
10,000 square links - - - -	1 chain.
25,000 square links - - - -	1 rood.
100,000 square links - - - -	1 acre.
10 square chains - - -	1 acre.
In flooring, roofing, painting, &c., 100 square feet make	1 square.
36 square yards of stone, brick, or slate	1 rood.

Pavers' and painters' work is computed in square yards.

3. *Solid or Cubic Measure.*

1728 solid inches make - - -	1 solid foot.
27 solid feet - - - - -	1 solid yard.

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40 feet of round timber, or 50 feet of hewn timber	- -	1 ton.
16 solid feet - - - - -		1 cord foot.
8 cord feet, or 128 solid feet - -		1 cord.

16½ solid feet make a perch of brick or stone.

4. *Table of Capacities.*

231 cubic inches make 1 wine gallon.

7276.5 solid inches make a wine barrel of 31½ gallons.

282 solid inches make 1 ale gallon.

10152 solid inches make 1 barrel of ale of 36 gallons.

277.274 cubic inches make an English imperial gallon, it being nearly one-fifth larger than the *wine*, and one-sixtieth smaller than the *ale* gallon.

268.8 solid inches make 1 gallon, dry measure.

2150.42 cubic inches make 1 bushel.

277¼ cubic inches make an English imperial gallon, dry measure.

2218.2 solid inches make an English imperial bushel.

2553.6 solid inches make a coal bushel in Vermont.

2688 solid inches make a coal bushel in New York.

14553 cubic inches make a hogshead of wine.

31020 solid inches make a hogshead of ale of 110 gallons.

15228 cubic inches make an ale hogshead of 54 gallons.

1 ounce Troy weight equals 1.097143 ounce avoirdupois.

7000 grains Troy weight make 1 pound avoirdupois.

144 pounds avoirdupois make 175 pounds Troy weight.

Plumbers' work is rated at so much a pound, or by the hundred.

¶ 13. MENSURATION.

Four methods of raising a perpendicular.

1. Take two inches between the legs of a pair of dividers, or compasses; then place one foot of the dividers on the end of

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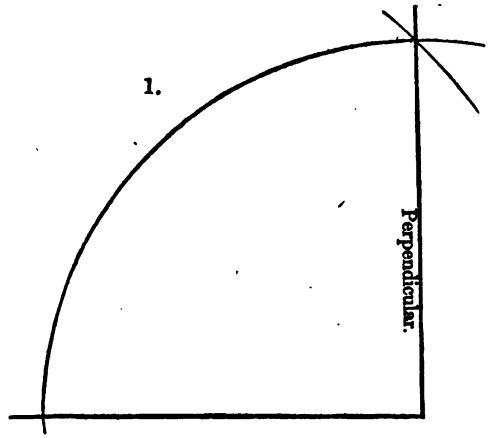
the line upon which you wish to erect the perpendicular, and strike an arc of a circle equal to, or greater than one-fourth of the circumference of the circle of which it is a part, and cutting the line on which the perpendicular is to be raised; then take two inches and thirteen-sixteenths of an inch between the legs of the dividers, and, placing one foot on the line where it is cut by the arc of the circle, strike the arc with the other leg, and the point where the two arcs cut each other will be perpendicular to the end of the line: consequently, draw the perpendicular from the end of the line through this point, and the thing is done. Thus—



2. Place one foot of the dividers on the end of the line, and strike a circular arc cutting the line; then with the same space between the dividers, place one foot on the line at the point where it is cut by the arc, and set off double the space between the dividers on the said arc; then place one foot of the dividers on the point in the arc to which the dividers reached the first step, and strike a circular arc over the point; then place one foot of the dividers on the point to which the dividers reached the second step, and strike a circular arc cutting the former, and the point where the arcs cut each other will be per-

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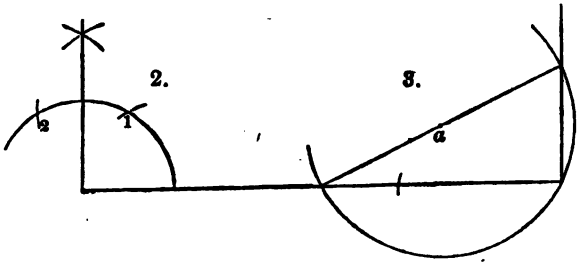
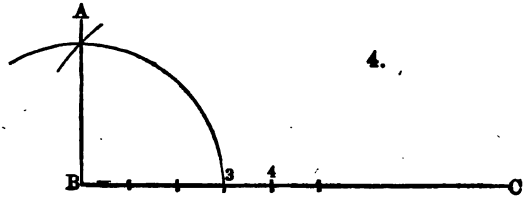
pendicular to the end of the line. In the same manner a perpendicular may be drawn from any point in the line. Thus—



3. Place one foot of the dividers on the end of the line, and the other at *a*, (some point between the line and the perpendicular;) then strike a circular arc cutting the line, and through the centre of the circle, of which the said arc is a part, draw a right line from the point where the arc cuts the given line, and extend it until it meets the said arc on the opposite side of the diameter; and the point where the right line meets the arc will be perpendicular to the end of the given line: consequently, draw the perpendicular through this point, and the thing is done.

4. Draw a right line and lay off, or set off, 5 equal spaces on this line; then, having taken 3 of these between the legs of the dividers, place one foot on the end of the line and strike a circular arc; then, having taken the 5 spaces between the legs of the dividers, place one foot on the 4th space from the end of the line, and strike an arc cutting the former, and the point where the two arcs cut each other will be perpendicular to the end of the line. Thus—



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ANGLES.

An angle is the space intercepted between two lines which intersect, or cross each other.

If one of the lines falls perpendicularly on the other, making an angle equal to 90 degrees, or the fourth part of the circumference of a circle, it is called a *right angle.* Thus the angle ABC, or the angle at B, in the above figure, is a right angle.

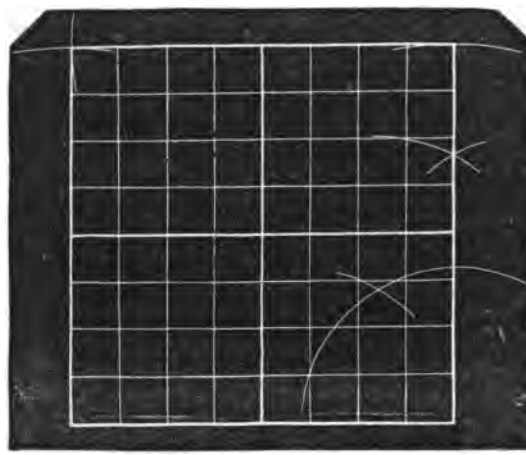
If the space included between two lines be less than a right angle, it is called an *acute angle;* but if the space be greater than a right angle, it is called an *obtuse angle.*

The rules for drawing the square, rectangle, rhombus, rhomboid, and trapezoid, are so simple, and so clearly illustrated by the figures, that it is deemed unnecessary to introduce them into this work.

¶ 14. MENSURATION OF SUPERFICIES.

The square is a figure bounded by four right and equal lines, and contains four right angles.

THE SQUARE.



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To find the area of the square:—

Multiply one side into itself, or square the given side.

EXAMPLES.

1. What is the area of a square whose side is 8 fourths of an inch? *Ans.* $\frac{64}{16}$ of a square inch, or 4 square inches.

The above *figure* is 2 inches on each side, or 8 fourths of an inch; and it is divided into four squares, (each of which is 1 inch on each side,) and also into 64 small squares, each of which is $\frac{1}{4}$ of an inch on each side; and since the square of $\frac{1}{4}$ is $\frac{1}{16}$, it is manifest that the above square contains $\frac{64}{16}$ of a square inch, or 4 square inches.

2. What is the area of a square 30 rods on each side? of one 40 rods on each side? of one 25 rods on each side? of one 15 rods? of one 12 rods? of one 10 rods? of one 6 rods? of one 80 rods? of one 120 rods? and of one 320 rods?

To solve the above by the sliding rule, set the slider as directed in ¶ 9, and over the number of rods on a side, found on the line D, will be found the number of square rods on the line C. Or, when the slider is in its usual position, that is, when the rule is shut, over the number of rods on a side of the given square, will be found the number of acres it contains on the C line. Thus, over 40 rods on the line D, at the right, we find 10 acres on the line C; and over 30 we find 5.6 acres; over 25 we find 3.9 nearly; over 20, 2.5; over 15, 1.41; over 12, 0.9; over 10, 0.625; over 6, 0.225; over 80, 40; over 120, 90; and over 320, we find 640 acres.

A square piece of land 10 chains on a side contains 10 acres; if, therefore, the side of a square piece of land be given in chains, *set* 10 on D under 10 on C, and over the number of chains on a side, found on D, will be found the number of acres on the line C.

If the side of a square be given in inches, to find its area in feet by the sliding rule:—Place 1 on C over 12 on D, and over the given side, found on D, will be found the number of square feet on the C line.

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3. What will be the value of a plot of ground 15 rods square, at $5 the square rod? *Ans.* $1,125.

To find the value of a square piece of land at so much per rod, or chain:—Place the price of one rod, or one chain, found on C, over 1 on D, (calling the 10 one,) then over the side of the piece found on D, will be found the answer required, on C.

4. What is the value of a square field 17 chains on a side, at 25 cents per square chain? *Ans.* $72.25.

5. What would 15 feet square of plastering cost, at $0.015 per square foot? *Ans.* $3.375.

To find the side of a square, the area being given:—Extract the square root of the given area. See ¶ 5 and 9.

To find the distance between the opposite corners of a square:—Extract the square root of twice the area.

6. A square field contains 90 acres of land; I require the length of its side, and the distance between its opposite corners. *Answers,* 120; and 169.7 rods.

In this example, before we extract the root by numbers, we must reduce the acres to rods; but it has been shown, that, when the slider is in its usual position, over any number of rods found on the line D, is the number of acres on the line C; and consequently, under the given number of acres found on C, will be found the number of rods on one side, viz. 120; and under twice the area, or 180 acres, will be found the distance between the opposite corners, viz. 169.7.

7. How many acres does a town contain which is 6 miles square? *Ans.* 23,040.

As 640 acres make one square mile, if we place 640 on the C line over 1 on the line D, then over 6 on D, will be found the number of acres on the line C.

Feet multiplied by feet produce feet.

Feet multiplied by inches, and the product divided by 12, give square feet.

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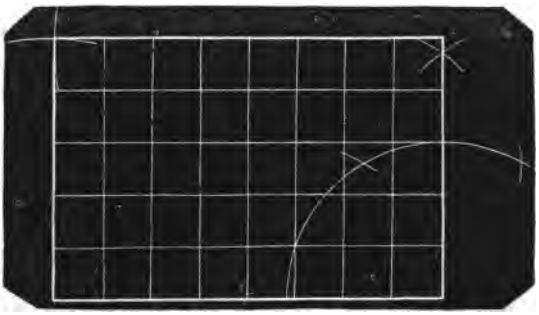
Inches multiplied by inches, and the product divided by 144, give feet.

Feet multiplied by primes, or lines, and divided by 144, give square feet.

Inches multiplied by lines, or primes, and divided by 12, give square inches; or inches multiplied by primes, and the product divided by 1728, give square feet.

Primes multiplied by primes, and the product divided by 144, give square inches.—A prime is one-twelfth of an inch, and a square prime is $\frac{1}{144}$ of a square inch.

¶ 15 RECTANGLE.



The rectangle is a figure bounded by four right lines, the opposite sides being equal and parallel, and its four angles right angles.

To find the area of a rectangle:—

Multiply the length by the breadth.

EXAMPLES.

1. What is the area of a board 8 inches long and 6 inches broad? *Ans.* 48 inches, or $\frac{1}{3}$ of a square foot.

2. What is the area of a piece of land 8 rods long and 5 rods broad? *Ans.* 40 square rods.

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The above plate represents such a piece of land; and the correctness of the rule may be demonstrated by counting the squares.

3. What is the area of a board 12 feet long and 10 inches broad? *Ans.* 10 feet.

Inches being twelfths of a foot, if we multiply the length in feet by the breadth in inches, the product will be twelfths of a square foot; and, consequently, to reduce the product to feet, divide it by 12. Or, (by the Rule of Three,) as 12 is to the length of the board in feet, so is its breadth in inches to its area in square feet. To find the area by the sliding rule, see ¶ 4.

4. What is the area of a board 13 feet in length and 15 inches in breadth? *Ans.* 16.25 feet.

By the sliding rule, *set* the length in feet found on the line B, (viz. 13,) under 12 on the line A, calling the 13 thirteen feet, and the 12 twelve inches; then under the width of any board in inches found on the line A, will be found its area in square feet on the line B, its length being 13 feet.

5. What are the contents, or areas, of the following boards, each being 14 feet in length, and one of them 14 inches, one 18, one 20, one 22, one 30, one 10, one 8, one 6, and one 4 inches in breadth? *Answers,* 16.3; 21; $23\frac{1}{3}$; $25\frac{3}{4}$; 35; $11\frac{3}{4}$; $9\frac{1}{3}$; 7; $4\frac{2}{3}$ feet.

Having *set* 14 on B under 12 on A, under the respective widths found on A will be found the areas on the line B.

6. How many square feet are there in the four sides of a room, 22 feet long, 17 broad, and 11 feet in height? And how much would it cost to paper the walls at three cents for a square foot? *Answers,* 858 feet; $25.74.

7. I desire to cut off one square foot from a board 30 inches broad; how far from the end of the board must I saw it off?

$144 \div 30 = Ans.$ 4.8 inches.

8. A piece of cloth 36 yards long contains 63 square yards; I require its breadth. *Ans.* $1\frac{3}{4}$ yards.

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9. How many panes of glass, each 8 by 10 inches, will be required for a window 5 feet in height, and 2 feet 8 inches broad? *Ans.* 24.

10. There is a house with 3 tiers of windows, and 3 windows in a tier; the height of the first tier is 7 feet 10 inches, the second is 6 feet 8 inches, and the third 5 feet 4 inches; and the breadth of each is 3 feet 11 inches; what will the glazing come to at 18¾ cents per square foot?

Ans. $43.6953.

11. The circumference, or perimeter of a room is 90 feet 11 inches, and its height 9 feet 8 inches; what is the area of its walls in square yards?

Ans. 97 yards, 5.87 feet, nearly.

To find the side of a square whose area shall equal that of a given rectangle:—

Extract the square root of the area of the rectangle.

Or, when the length and breadth of the rectangle are given, to find the side of an equal square by the sliding rule:—

Set one side of the given rectangle, found on the C line, over the same number found on the line D; then find the other side on the C line, and under it will be found the side of an equal square on the line D.

EXAMPLES.

1. The sides of a rectangle are 40 and 10; I require the side of an equal square. *Ans.* 20.

Set 10 on C over 10 on D, and under 40 found on C, will be found 20, the side of the equal square, on the D line.

2. What is the side of a square equal to a rectangle 25 by 9 feet? *Ans.* 15 feet.

3. What is the side of a square equal to a rectangle 45 by 80 rods? *Ans.* 60 rods.

Having found the side of an equal square by the sliding rule, the area of the rectangle may be found in acres, or feet,

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by the rules given in ¶ 14. Thus, in the above example, having found the side of the square, 60 rods, *shut* the slider, and over 60 rods found on D, we find the area, viz. 22.5 acres.

4. What is the side of a square equal to a rectangle whose sides are 32 by 24 rods? And what is its area?

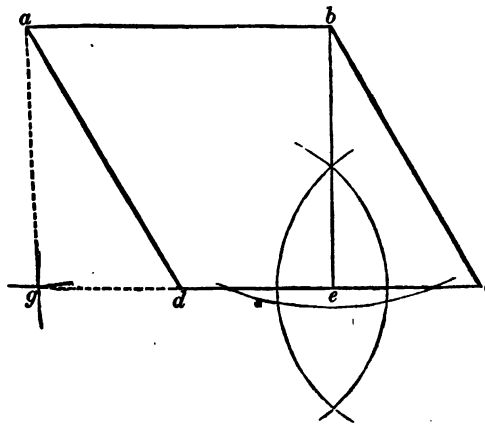
Answers, 27.75 rods, and 4.8 acres.

5. The length of a rectangular pavement is 47 feet 9 inches, and its breadth 13 feet 11 inches; how many stones, each 17 inches long and $10\frac{1}{2}$ inches broad, will serve to pave it?

Ans. 536.

6. The length of a house is 40 feet 9 inches, and the sloping height of the roof above the walls is 19 feet 5 inches; how many slates will cover the roof, supposing each slate to cover $17\frac{1}{2}$ square inches? *Ans.* $1302\frac{13}{35}$.

¶ 16. RHOMBUS.



The rhombus is a figure bounded by four right and equal lines, and has two of its angles obtuse, and two acute, the opposite angles being equal, and its opposite sides parallel.

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The area of a rhombus is equal to that of a rectangle, one side of which is equal to one side of the rhombus, and the other equal to its perpendicular breadth; the rhombus *abcd* being equal to the rectangle *abeg*, and the triangle *bec* being equal to the triangle *adg*.

Therefore, to find the area of a rhombus:-

Multiply the length of one side by its perpendicular breadth.

EXAMPLES.

1. What is the area of a rhombus, one side of which is 6 chains and 20 links, and whose perpendicular breadth is 5 chains and 45 links? *Ans.* 3 acres, 1 rood, and 20 rods.

2. What is the area of a rhombus, whose side is 40, and perpendicular breadth 32 rods? *Ans.* 8 acres.

By the sliding rule we find the side of an equal square to be 35.75 rods, and the area 8 acres.

3. How many square feet does a rhombus contain, whose side is 40 inches, and whose perpendicular breadth is 22.5 inches? *Ans.* 6.25 feet.

By the rule, we find the side of an equal square to be 30 inches; then placing 1 on C over 12 on D, (calling the 1 one foot and the 12 twelve inches,) and over 30 inches will be found the area in feet, viz. 6.25.

4. What is the area of a rhombus, whose side is 13 feet and perpendicular breadth 32 inches? *Ans.* $34\frac{1}{2}$ feet.

5. How many square yards does a rhombus contain, whose side is 37 feet 10 inches, and the perpendicular height 28 feet 9 inches? *Ans.* 120 yards, 7 feet, 102 inches.

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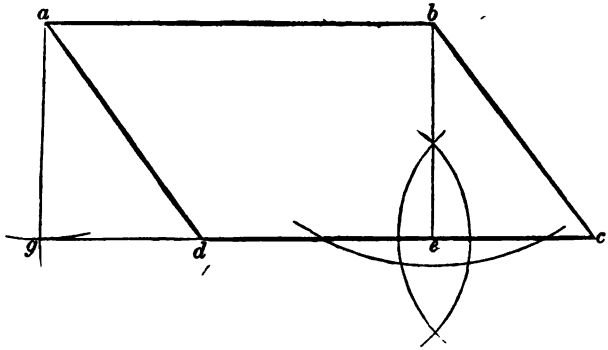
¶ 17. RHOMBOID.



The rhomboid is a figure bounded by four right lines, having its opposite sides equal and parallel, and two of its angles obtuse and two acute, the opposite angles being equal.

The area of the rhomboid is equal to that of a rectangle whose length is equal to the longest side of the rhomboid, and its breadth equal to the perpendicular breadth of the rhomboid.

This is evident from the figure, the triangle *bec* being equal to the triangle *agd*, and the rectangle *abeg* being equal to the rhomboid *abcd*.

Therefore, to find the area of a rhomboid :—

Multiply the longest side by its perpendicular breadth.

EXAMPLES.

1. What is the area of a rhomboid, whose length is 37 feet, and perpendicular breadth 5 feet 3 inches ?

Ans. 194.25 feet.

2. What is the area of a rhomboid 17 feet in length and 22 inches in perpendicular breadth ? *Ans.* 31.2 feet, nearly.

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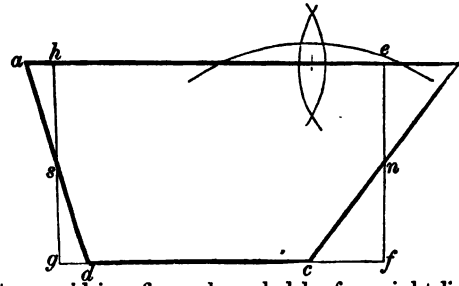
¶ 18. TRAPEZOID.



The trapezoid is a figure bounded by four right lines, having two of its sides parallel, but of unequal length.

By inspecting the above figure, it will be seen that the trapezoid, *abcd*, is equal to a rectangle, one of whose sides is equal to half the sum of the two parallel sides of the trapezoid, and the other equal to the perpendicular distance between these two sides. For, since *eb* is equal to *cf*, and *ah* is equal to *gd*, it is evident that *gf* or *he* is equal to half the sum of $ab+cd$; and since the triangle *ebn* is equal to the triangle *nfc*, and the triangle *ahs* is equal to the triangle *sgd*, it is manifest that the rectangle *hefg* is equal to the trapezoid *abcd*. But *hg* is the perpendicular between the two parallel sides of the trapezoid; and, consequently,

To find the area of the trapezoid:—

Multiply half the sum of the parallel sides by the perpendicular distance between them.

EXAMPLES.

1. The parallel sides of a trapezoid are 750 and 1,225 links, and the perpendicular distance between them is 1,540 links; what is its area? *Ans.* 15 acres, 33 rods.

2. How many square feet are there in a board 12.5 feet in length, and 15 inches wide at one end and 11 inches at the other, the ends being parallel? *Ans.* 13.54 feet.

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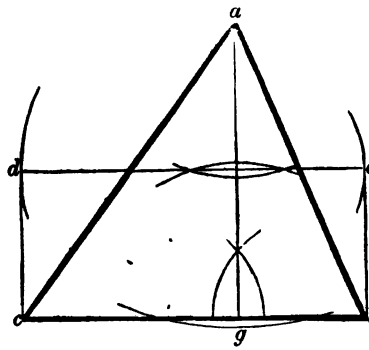
¶ 19. TRIANGLE.



The triangle is a figure bounded by three right lines, and contains three angles, the sum of which is always equal to two right angles, or 180 degrees.

When a triangle has all its sides equal, it is called an *equilateral* triangle, or a *trigon;* when two of its sides are equal, it is called an *isosceles* triangle; and when no two sides are equal, it is called a *scalēne* triangle, except it contains a right angle, when it is usually called a *right-angled* triangle. Any triangle is equal to a rectangle, one side of which is equal to the base, and the other equal to half the perpendicular height (or half the altitude) of the triangle.

Therefore, to find the area:—

Multiply the base of the triangle by half of its perpendicular altitude.

EXAMPLES.

1. What is the area of a triangle, whose base is 625 links, and its perpendicular altitude 520 links? *Ans.* 1.625 acres.

2. How many square yards are there in a triangle whose base is 40, and its perpendicular height 30 feet?

Ans. 66⅔ yards.

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3. How many acres does a triangle contain, whose base is 40, and its perpendicular height 30 rods?

Ans. 3.75 acres.

To solve this example by the sliding rule, first find the side of an equal square, as directed under the rectangle, which in this example we find to be 24.5 rods nearly; then *shut* the slider, and over 24.5 rods, found on the line D, we find 3.75 acres for the answer.

To find the altitude of an equilateral triangle, its side being given:—

From the square of the given side subtract one-fourth of the square of the given side, and extract the square root of the remainder.

4. What is the altitude, and what is the area of an equilateral triangle whose side is 1?

The altitude is .86602540378444, and its area is .43301270189222.

Trigons, or equilateral triangles, are to each other as the squares of their sides. Therefore, to find the area of an equilateral triangle, its side being given:—

Multiply the square of the given side by the decimal .4330127.

5. What is the area of an equilateral triangle whose side is 34? *Ans.* 500.5, nearly.

To find the side of an equilateral triangle whose area shall be 1:—

Divide unity, or 1, *by the decimal* .4330127, *and the quotient is the area of a square drawn on one side of the triangle,* [viz. 2.3094, very nearly,] *the square root of which is the side of the triangle,* viz. 1.51967, nearly.

By the above rule, the side of any equilateral triangle may be found, its area being given; or, multiply the side of a square by 1.51967, and the product will be the side of a trigon of equal area.

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6. What is the side of an equilateral triangle whose area is 144 square inches? *Ans.* 18.236 inches.

7. What is the side of an equilateral triangle whose area is 1 acre? *Ans.* 19.225 rods.

Since the areas of trigons are as the squares of their sides, and since the numbers on the line C are the squares of those on the line D, if we place 1 on C over 18.236 on D, over the side of any equilateral triangle on D, (its side being given in inches,) will be found the number of feet on C. Or, *set* 1 on C over 19.225 on D, and over the side of any equilateral triangle given in rods, will be found its area in acres.

8. The side of a trigon is 31 inches; its area in feet is required. *Ans.* 2.9 feet, nearly.

9. What is the area of an equilateral triangle whose side is 38 rods? *Ans.* 3.9 acres, nearly.

10. What is the area of a trigon whose side is 45 rods? *Ans.* 5.48 acres.

When the sides of a scalēne triangle are given, to find its area:—

From half the sum of the three sides subtract each side severally; then multiply together the half sum of the three sides, and the three remainders, and extract the square root of the product for the area.

11. What is the area of a triangle whose sides are 20 and 30 and 40 rods? *Ans.* 1.8159 acre.

The half sum of the three sides is 45, and the three remainders 25 and 15 and 5; and $45 \times 25 \times 15 \times 5 = 84375$, the square root of which is 290.473 rods = to 1 acre, 3 roods, and 10 rods.

12. If the sides of a triangle are 134 and 108 and 80 rods, what is its area? *Ans.* 4319.9 square rods.

Having the three sides of any triangle given, to find the point in the base, or longest side of the triangle, where a perpendicular let fall from the opposite angle will meet the base, or longest side, say:—As the longest side is to the sum of the other two sides, so is their difference to the difference of the

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segments formed on the base by the perpendicular drawn from the opposite angle; then add half of this difference to half the sum of the two segments, (that is, to half of the longest side,) and it will give the greater segment; and subtract the said half difference from the said half sum, and it will give the less segment.

It is not necessary that the longest side should always be taken for the base of the triangle, but the rule may be applied in any case, when the perpendicular drawn from the angle opposite the side chosen for a base, does not fall without the triangle, or beyond the base.

13. Suppose the lower ends of two rafters are 42 feet apart, and that one of the rafters is 36 feet in length, and the other 32; the height of the ridge, above the plates on which the rafters rest, is required. As 42 : 68 : : 4 to the difference of the segments, viz., 6.4762, and $\frac{6.4762}{2}+\frac{42}{2}=24.2381$ the greater segment, [as *cg*, see the figure,] and $\frac{42}{2}-\frac{6.4762}{2}=17.7619$, the less segment. Having found the segments, if we square the length of either of the rafters, and subtract from its square the square of the segment under that rafter, the remainder will be the square of the height of the ridge, the square root of which will give the required height. Thus, $(36\times36)-(24.2381\times24.2381)=708.51450839$, the square of the height of the ridge, the square root of which is 26.61773 feet, the required height.—A roof is said to have a *true pitch* when the length of the rafters is $\frac{3}{4}$ the breadth of the building.

14. The sides of a triangle are 20, 28, and 31 rods. What is the length of a perpendicular falling from the opposite angle upon the longest side? *Ans.* 17.703 rods, very nearly.

Similar triangles (that is, triangles whose angles are equal) *are to each other as the squares of their corresponding sides*, or *as the squares of their altitudes;* and this rule, or statement, is true in respect to all similar superficies.

15. If the sides of a triangle are 42, 36, and 32 rods, at what distance from the apex of the triangle must a line be

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drawn parallel to the longest side, so as to cut off $\frac{1}{3}$ of the area? And what will be the length of the base of the part cut off?

Answers, 15.36788 rods; and base, 24.24871 rods.

The square root of $\frac{1}{3}$ of the square of the altitude of the given triangle will be the altitude of the part cut off; and the square root of $\frac{1}{3}$ of the square of the given base will be the base of the part cut off.

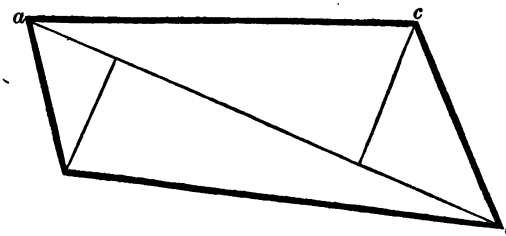
To find the area of a triangle when two of its sides and the contained angle are given:—

Multiply the NATURAL SINE *of the contained angle by the product of the two sides into each other, and half this product will be the area.* (See the table, ¶ 73.)

Required the number of square yards in a triangle, two of whose sides are 50 feet and 42 feet 6 inches, and the contained angle 45 degrees? *Ans.* 83.47788 yards.

How many square yards are contained in a triangle, two of whose sides are 42½ and 75 yards, and the included angle 50 degrees? *Ans.* 1221 yards.

¶ 20. TRAPEZIUM.



A trapezium is a figure bounded by four right lines, having no two sides equal, and no two angles which contain the same number of degrees.

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To find the area of the trapezium :—

Divide it into two triangles by a diagonal line, (as *ab* in the figure,) *to which draw perpendiculars from the opposite angles; measure the length of the diagonal and the two perpendiculars, and multiply the diagonal by the sum of the perpendiculars, and half the product is the area.* Or, Find the areas of the two triangles as directed in ¶ 19, and the sum of their areas will be the area of the trapezium.

EXAMPLES.

1. What is the area of a trapezium, whose diagonal is 42, and the perpendiculars falling upon it from the opposite angles 16 and 18 ? *Ans.* 714.

2. How many square yards of paving are there in a trapezium whose diagonal is 65, and the perpendiculars falling upon it 28 and 32.5 feet ? *Ans.* 222.08 yards.

When a trapezium can be circumscribed by a circle, and its four angles made to touch the circumference, its area may be found by the following rule.

From ½ the sum of the four sides subtract each side, severally ; then multiply together the four remainders and extract the square root of their product.

3. The sides of a trapezium inscribed in a circle, are 10, 12, 15, and 21 rods ; its area is required. *Ans.* 190.2 rods.

4. The four sides of a trapezium in a circle, are 600, 650, 700, and 750 links ; its area is required.

Ans. 4 acres, 2 roods, 4 rods.

POLYGONS.

Any figure bounded by more than four right lines, is called a *polygon ;* and if the sides are of equal length, it is called a *regular polygon ;* but if the sides are of unequal length, it is called an *irregular polygon.* To find the area of an irregular polygon :—

Divide it into triangles ; measure each of the triangles sepa-

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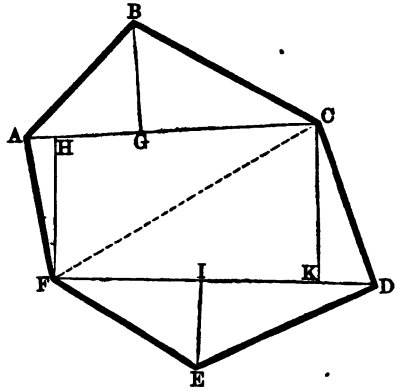
rately and find its area, and the sum of all the triangles will be the area of the polygon.

Find the area of an hexagonal figure from these measurements:—

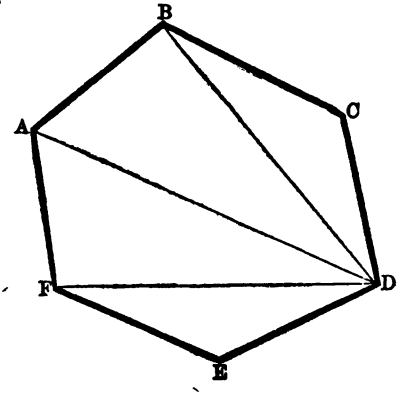


AC=525 links, BG=160 links, DF=490 links, FH=210 links, EI=100 links, CK=300 links.

Ans. 1 acre, 3 roods, 32.2 rods.

The figure, it is evident, is made up of the two trapeziums ABCF, and CDEF.

Required the area of the irregular hexagon ABCDEF from these data:—



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The side AB=690 links, BC=870 links, CD=770 links, DE=620 links, EF=770 links, AF=630 links, the diagonal FD=1210 links, the diagonal AD=1634 links, and the diagonal BD=1486 links. *Ans.* 12 acres, 3 roods, 37.19 poles.

In this example the polygon is divided into triangles of which the three sides are known; and hence their areas may be found by the rules for the triangle. See ¶ 19, under problem 10.

The sum of all the angles of any polygon is equal to twice as many right angles, wanting four, as the figure has sides; and when the polygon is regular, if the sum of the angles be divided by the number of angles, the quotient will be the value of one of the angles. Thus—

$10\times 90^\circ-(4\times 90^\circ)=900^\circ-360^\circ=540^\circ$, the sum of the angles of a pentagon; and $\frac{540^\circ}{5}=108^\circ$, the interior angle of a regular polygon of 5 sides.

¶ 21. REGULAR POLYGONS.

A regular polygon has its sides and angles equal; and its area is equal to that of a rectangle, one side of which is equal to the perimeter, or sum of all the sides of the polygon, and the other to one-half of the radius of the greatest inscribed circle, or to half the distance from the centre of the polygon to the middle of one of its sides. Therefore, to find the area of any regular polygon:—

Multiply the perimeter of the polygon by half the radius of the greatest inscribed circle.

A regular polygon of six sides is called a *hexagon;* one having five sides is called a *pentagon;* one having seven sides is called a *heptagon;* one having eight sides, an *octagon;* one having nine sides, a *nonagon;* one having ten sides, a *decagon;*

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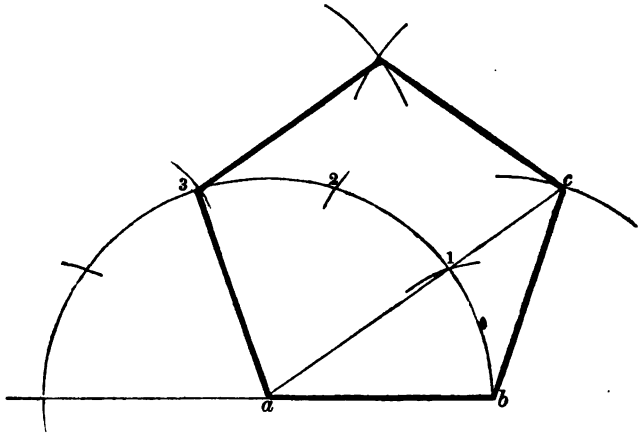
one having eleven sides, an *undecagon;* and one having twelve sides is called a *dodecagon.*

PENTAGON.



To draw a pentagon:—Draw the side *ab* of any required length; then take the side *ab* between the dividers, and having set one foot on the end of the line, at *a*, strike a semicircle, and divide the arc into five equal parts; then from the end of the line at *a*, draw a line through the first space from *b*, and extend it indefinitely; then with the side *ab* between the dividers, place one foot on the end of the line at *b*, and with the other mark the line *ac;* then place one foot of the dividers on the point found in the line *ac* at *c*, and strike an arc of a circle over the base of the polygon; then place one foot of the dividers on the third space from *b* on the arc of the semicircle, and strike an arc over the base of the polygon cutting the arc struck from the point *c*, and where the two arcs meet will be the vertex of the pentagon; then through the points found as described above, draw the sides of the pentagon.

To find the diagonal of any pentagon:—Multiply the given side by 1.618036. To find the perpendicular altitude of a pentagon:—Multiply the given side by 1.538842. To find the

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apothegm of a pentagon, that is, the distance from its centre to the centre of one of its sides:—Multiply the given side by .688191. To find the radius of the least circumscribing circle:—Multiply the given side of the pentagon by .850651.

What is the area of a pentagon whose side is 1?

Ans. 1.7204774.

Any regular polygon may be divided into as many equal triangles as the polygon has sides, by drawing lines from its centre to its several angles; hence it is manifest that we may find its area by multiplying its apothegm by half the perimeter of the polygon; or, by multiplying its perimeter by half its apothegm; that is, by half the perpendicular altitude of one of the several triangles which together make up the polygon.

The area of all regular polygons are as the squares of their sides. Therefore, *if we multiply the square of the side of any regular polygon by the area of a similar polygon whose side is* 1, *it will give the area.*

The following table exhibits the areas and apothegms of all regular polygons of not more than 12 sides, when the side is unity:

Name of Polygon.	Apothegm when the side = 1.	Area when the side = 1.	Angle.
Trigon . .	0.2886751	0.4330127	60°
Pentagon .	0.6881910	1.7204774	108°
Hexagon .	0.8660254	2.5980762	120°
Heptagon .	1.0382607	3.6339124	128° 34′$\frac{2}{7}$
Octagon . .	1.2071068	4.8284271	135°
Nonagon .	1.3737387	6.1818242	140°
Decagon .	1.5388418	7.6942088	144°
Undecagon .	1.7028436	9.3656399	147° 16′$\frac{4}{11}$
Dodecagon .	1.8660254	11.1961524	150°

EXAMPLES.

1. What is the diagonal of a pentagon, whose side is 7?

Ans. 11.326252.

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2. What is the apothegm of a pentagon, whose side is 10? *Ans.* 6.88191.

3. What is the perpendicular altitude of a pentagon, whose side is 20? *Ans.* 30.776840.

4. What is the area of a pentagon, whose side is 30 feet? *Ans.* 1548.42966 feet.

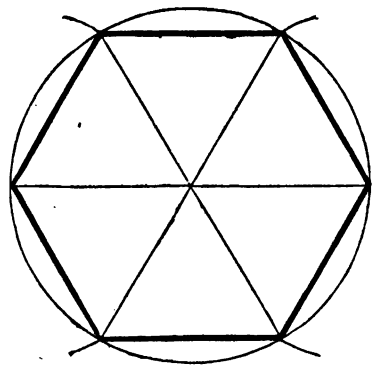
5. What is the number of square yards in a heptagon, whose side is 20 yards? *Ans.* 1453.56 yards.

6. How many acres are contained in an octagon, whose side is 5 chains? *Ans.* 12 acres, 0 roods, 11.4 rods.

7. How many square yards are contained in a decagon, whose side is 12 feet? *Ans.* 123.1072 yards.

8. How many acres are contained in a decagon, whose side is 2,050 links? *Ans.* 323 acres, 1 rood, 15.86 rods.

HEXAGON.



The side of a hexagon is equal to the radius of the least circumscribing circle. Therefore, to draw a hexagon, take the side of the hexagon between the dividers, and describe a circle; then, with the same space between the dividers, space round the circle, dividing it into six equal parts; then join the points thus found by right lines, and the polygon will be completed.

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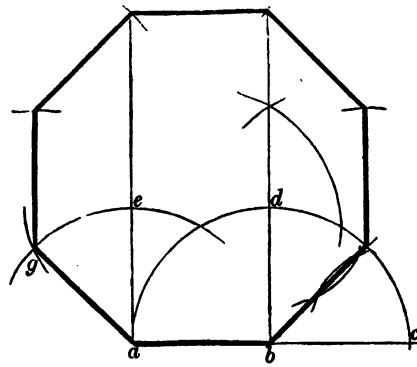
The hexagon may be divided into six equal trigons, by drawing lines from its centre to each of its angles, as may be seen in the figure; consequently, its area, its side being 1, will be equal to 6 times the area of an equilateral triangle, whose side is 1. And since we have shown, under the triangle, that the area of an equilateral triangle, whose side is 1, is 0.4330127, that of the hexagon will evidently be 6×0.4330127, or 2.5980762, the area laid down in the table.

OCTAGON.



To draw an octagon:—Draw the base, *ab*, of any given length, and erect perpendiculars on each end of the base, and extend them indefinitely. Extend the base, *ab*, to *c*, making *bc* equal to *ab*; then with *bc* for radius, place one foot of the dividers on the end of the base at *b*, and strike the arc *dc*; then divide the arc *dc* into two equal parts, and to the centre of said arc, draw a right line from the end of the base at *b*, and it will be one side of the octagon; then from the other end of the base, with *ab* between the dividers, strike the arc *eg*; and from *e*, where said arc cuts the perpendicular, set off *eg* equal to half of the arc *dc*, and draw *ag* for the third side of the octagon; then from the ends of the two sides last drawn, draw two sides parallel to each other, and likewise parallel to the perpendiculars raised on each end of the base; then, having drawn these two sides as directed, take *ab*, the base, between the dividers, and from

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the upper ends of these two sides strike arcs cutting the two perpendiculars; join these two points with each other, and with the ends of the two sides last drawn, and the figure will be complete.

To find the radius of a circle which will circumscribe an octagon:—Find its apothegm, and add the square of its apothegm to the square of half the side of the octagon, and extract the square root of the sum.

Carpenters and millwrights not unfrequently wish to cut an octagon from a square, and to enable them to do this with facility, there are two lines laid down on the sliding rule, one of these lines having an M and the other an E standing on the left. The M signifies *middle*, the numbers on this line denoting the side of a square, being laid down just as far from the end of the rule on the right, as it is necessary to cut from the middle of the square in order to throw it into an octagon. For example, 12 on this line, will be found 2.485 inches, or $2\frac{1}{2}$ inches nearly, from the end of the rule; and consequently, if you wish to cut an octagon from a square 12 inches on a side, you must cut the square 2.485 inches each way from the middle. You will find 18 on this line $3\frac{3}{4}$ inches from the end of the rule; therefore, if the square be 18 inches on a side, to throw it into an octagon, you must cut the square $3\frac{3}{4}$ inches from the middle of each side.

The E on the left of the other line, signifies *edge;* and the numbers on this line, denoting the side of a square, are placed just as far from the end of the rule, as it is necessary to cut from the edge of the given square to throw it into an octagon.

Thus, you will find 12 on this line 3.515 inches from the end of the rule, which signifies, that, if a square be 12 inches on a side, you must cut 3.515 inches from its edge in order to form an octagon.

The above lines seldom give the side of a square more than 36 inches on a side; but should you be required to cut an octagon from a square 40, or 50, or more inches on a side, the distance which you must cut from the middle of the side may be found as follows:—Place 2.485 inches on the line B, under 12 on the line A; then find the side of the square on the line

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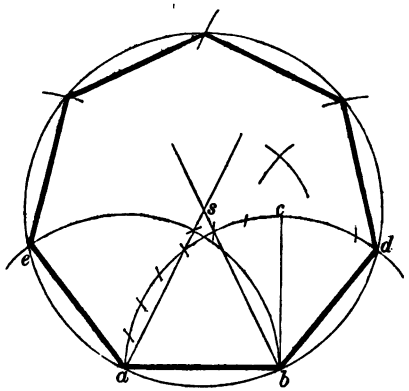
A, and under it will be found the distance on B, which it will be necessary to cut from the middle of the side of the given square, to throw it into an octagon. Or, place 3.515 inches on B under 12 on A, and under the side of the given square will be found the distance which you must cut from the edge of the square to form an octagon. Or, place 4.97 inches on B, under 12 on A, and under the side of any square found on A, will be found the side of the octagon cut from it; or, over the side of the octagon found on B, will be found the side of a square from which it may be cut, on the line A. Having set 4.97 on B under 12 on A, under 40 on A you will find 16.6, which is the side of an octagon cut from a square whose side is 40; and under 60 you will find 24.8, which is the side of an octagon cut from a square whose side is 60; and over 20 you will find 48.15, which is the side of a square from which you may cut an octagon whose side is 20.

RULE FOR DRAWING A POLYGON OF ANY NUMBER OF SIDES.

HEPTAGON.



To draw a polygon of any number of sides:—Draw one side of any required length, as *ab* in the figure; then place one foot of the dividers on the end of the side at *b*, and having extended the other foot to *a*, strike the arc *acd*; then, placing

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one foot of the dividers on the end of the base at *a*, strike the arc *be*; then, from the end of the base at *b*, erect the perpendicular *bc*, and divide the arc *ac* into as many equal parts as the polygon has sides: if you are drawing a heptagon, divide the arc *ac* into seven equal parts; but if you are drawing an octagon, divide the said arc into eight equal parts; if a nonagon, divide the arc into nine equal parts, &c. Then draw a right line from the end of the base at *b*, through the second space on the arc, from *c*, and extend it indefinitely; then place one foot of the dividers on the end of the base at *a*, and extend the other foot till it reaches the second space from *c*, and with this space between the dividers, place one foot on the end of the base at *b*, and set off this distance on the arc *be*; then from the end of the base at *a*, draw a right line through the point found in the arc *be*, and extend it till it cuts the line drawn from *b* through the second space from *c*, and the point where these two lines cut each other, will be the centre of the circle in which the given polygon must be drawn: therefore, place one foot of the dividers on this point, and extend the other foot till it reaches the end of the base at *a* or *b*, and with this distance for radius describe a circle; then, with the base *ab* between the dividers, space round the circumference, and it will divide it into as many equal parts as the polygon has sides; join the points thus found in the circumference, and the figure will be completed.

To find the perpendicular altitude of an octagon, or the side of its circumscribing square, (without referring to the table:)—

Square one side of the octagon, and extract the square root of half the square, and add double the said root to the side of the octagon, and the sum will be its perpendicular altitude, or the side of its circumscribing square.

To find the area of an octagon:—

Square its perpendicular altitude, and from its square subtract twice the square of one of its sides.

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EXAMPLES.

1. What is the area of a pentagon, whose side is 95, and its apothegm 65.36? *Ans.* 15523.

2. What is the side of a pentagon whose area is 1?

Ans. 0.7623, nearly.

Solution, $\frac{1}{1.7204774} = 0.5812$ the square of the side of a pentagon whose area is 1, and $\sqrt{0.5812} = .7623$, the required side.

To find the area of a pentagon by the sliding rule:—Place 1 on C over .7623 on the line D, and over the side of any pentagon found on D, will be found its area on the line C; or, if the side of a pentagon be given in inches and decimal parts of an inch, and its area is required in feet, (because 9.15 inches is the side of a pentagon which contains one square foot,) place 1 on C over 9.15 on D, and over the side in inches found on D, will be found the area in feet on the line C.

3. How many square feet in a pentagon 10 inches on a side? in one $14\frac{1}{2}$ inches on a side? in one 20 inches on a side? in one 25 inches on a side? in one 30 inches on a side? in one 35 inches on a side? and in one 40 inches on a side?

Answers in order,—1.18; 2.51; 4.75; 7.45; 10.7; 14.62; 19.16 square feet.

4. What is the side of a hexagon whose area is 1?

Ans. 0.621, nearly.

Solution, $\frac{1}{2.598907}$ = the square of the required side, the square root of which is 0.621, the required side.

5. What is the side of a hexagon whose area is one acre, or 160 square rods? *Ans.* 7.861 rods.

6. How many acres in a hexagon whose side is 12 rods? in one whose side is 20 rods? and in one whose side is 60 rods?

Solution by the sliding rule:—Place 1 on C over 7.86 on D, and over 12 rods on D we find 2.32 acres; and over 20 we find 6.45 acres; and over 60 we find 58.45 acres.

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7. What is the side of an octagon whose area is 1?
Ans. 0.45509.

8. What is the side of an octagon whose area is 144 square inches? *Ans.* 5.46 inches.

9. What is the side of an octagon whose area is one acre?
Ans. 5.76 rods, nearly.

10. How many square feet in an octagon whose side is 6 inches? in one whose side is 10 inches? in one whose side is 15 inches? and in one whose side is 20 inches?

Solution.—Place 1 on C over 5.46 on D, and over the side in inches found on D, will be found the areas in feet on the line C. *Answers* in order,—1.21; 3.33; 7.52; 13.25 feet.

11. How many acres in an octagon, whose side is 5.9 rods? in one whose side is 8 rods? in one whose side is 15 rods? in one whose side is 30 rods? and in one whose side is 40 rods?
Answers in order,—1.047 acres; 1.925 acres; 6.76 acres; 27.1 acres; and 48 acres.

12. How many square yards in a decagon, whose side is 12 feet? *Ans.* 123.1072 yards.

¶ 22. CIRCLE.

The circle is a plane figure bounded by a curved line, every point or part of which is equally distant from a certain point within called the centre. The circumference, or bounding line, is sometimes called a circle.

By examining the figures representing the regular polygons, it will be seen, that they may be divided into as many triangles as the polygon has sides; and that, consequently, their areas may be found by multiplying the perimeter of the polygon by half the apothegm, or half the radius of the greatest inscribed circle. Now if a polygon of a great number of sides be drawn in a circle, its perimeter will evidently approach very near to

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the circumference of the circle; and if we should suppose the number of sides of the inscribed polygon to be absolutely infinite, its perimeter would evidently coincide with the circumference of the circle: the circumference of the circle may, therefore, be considered the perimeter of a polygon of an infinite number of sides; and hence it may be regarded as the sum of the bases of an infinite number of triangles, whose vertexes all meet at the centre of the circle. Therefore—

To find the area of a circle:—

Multiply the circumference by half the radius, or one-fourth of the diameter.

The exact ratio of the circumference of the circle to its diameter, though for a long time a celebrated problem with geometricians, never has been and never can be exactly obtained; yet we may easily find the ratio of the diameter of a circle to its circumference sufficiently near for any practical or scientific purpose.

There are various methods of approximating towards the circumference of the circle. The method of approximation adopted by the ancients was as follows. They supposed a polygon of a great number of sides to be inscribed in the circle, and a polygon of the same number of sides to be circumscribed around the circle; they then, by a plain but laborious method, calculated the perimeters of the inscribed and circumscribed polygons, and took the arithmetical mean between them for the true circumference. By this method, the indefatigable Ludolphus Van Ceulen, a Dutch mathematician, (who died in 1610,) calculated the ratio of the diameter of a circle to its circumference, true to thirty-five decimal places.

Thus, calling the diameter 1, he found the circumference to be 3.14159265358979323846264338327950288.

That prince of mathematicians, Sir Isaac Newton, discovered a much more rapid method of approximation, which we will give for the gratification of those who are curious in such matters.

As we have shown, under the hexagon, ¶ 21, the chord of 60° (that is, one side of the hexagon) equals radius, (or the

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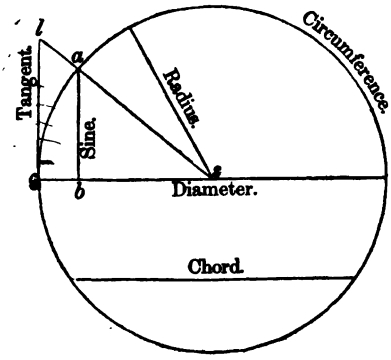
CIRCLE.



semidiameter of the circle,) hence the sine of 30° (that is, half the chord of 60°) equals half of radius; and if ab, the sine of 30°, be represented by x, and radius by r, then ac, the arc of 30°, will $= x + \frac{x^3}{6r^2} + \frac{3x^5}{40r^4} + \frac{5x^7}{112r^6} + \frac{35x^9}{1152r^8} + \frac{63x^{11}}{2816r^{10}}$, &c., in which series the coefficients, $\frac{1}{6}$; $\frac{3}{40}$; $\frac{5}{112}$, &c., are found thus: $\frac{1\times1}{2\times3}=\frac{1}{6}$; and $\frac{1}{6}\times\frac{3\times3}{4\times5}=\frac{3}{40}$; and $\frac{3}{40}\times\frac{5\times5}{6\times7}=\frac{5}{112}$; and $\frac{5}{112}\times\frac{7\times7}{8\times9}=\frac{35}{1152}$; and $\frac{35}{1152}\times\frac{9\times9}{10\times11}=\frac{63}{2816}$; and $\frac{63}{2816}\times\frac{11\times11}{12\times13}=\frac{231}{13312}$; and $\frac{231}{13312}\times\frac{13\times13}{14\times15}=\frac{39039}{2795520}=\frac{143}{10240}$; and $\frac{143}{10240}\times\frac{15\times15}{16\times17}=\frac{32175}{2785280}=\frac{6435}{557056}$, &c. Now, if radius, or r, $= 1$, then ab, the sine of 30°, represented in the series by x, will be $\frac{1}{2}$; and therefore the series may be expressed thus: $\frac{1}{2}+\frac{1}{6}\times\left(\frac{1}{2}\right)^3+\frac{3}{40}\times\left(\frac{1}{2}\right)^5+\frac{5}{112}\times\left(\frac{1}{2}\right)^7+\frac{35}{1152}\times\left(\frac{1}{2}\right)^9$, &c. Therefore the arc $ac = \frac{1}{2}+\frac{1}{48}+\frac{3}{1280}+\frac{5}{14336}+\frac{35}{589824}+\frac{63}{5767168}+\frac{231}{109051904}+\frac{143}{335544320}+$

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$\frac{6435}{73014444032}$, &c., which, expressed in a decimal form, will stand thus:—

0.500000000000000
0.020833333333333
0.002343750000000
0.000348772321429
0.000059339735243
0.000010923905807
0.000002118257376
0.000000426178211
0.000000088133247
0.000000018618793
0.000000004000825
0.000000000871722
0.000000000192143
0.000000000042767
0.000000000009599
0.000000000002171
0.000000000000494
0.000000000000114
0.000000000000026
0.000000000000006

The arc ac = 0.523593775598296

Since 30° is $\frac{1}{12}$ of 360°, the arc of 30° will be $\frac{1}{12}$ of the circumference of the circle of which it is a part; and since we have calculated the arc of a circle whose diameter is 2, if we multiply the said arc by 6, it will give the circumference of a circle whose diameter is 1. Thus:—

.523598775598296
6
3.141592653589776*

* The arc of one second = .0000048481368111, *radius being* 1.
The arc of one minute = .0002908882086657, *radius being* 1.
The arc of one degree = .017453292519942, *radius being* 1.
The sine of one degree = .017452408, *radius being* 1.
The tangent of one degree = .017455066857, *radius being* 1.

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Thus we find the ratio of the diameter to the circumference to be as 1 to $3\frac{1}{7}$ nearly; or more nearly, as 1 to 3.1416. Therefore, where great accuracy is not required, to find the circumference of a circle, its diameter being given:—

Multiply the diameter by 3.1416. Or, the circumference being given, to find the diameter:—

Divide the circumference by 3.1416.

EXAMPLES.

1. If the earth's equatorial diameter be 7,930 miles, what is the circumference of the earth measured on the equinoctial line? *Ans.* 24913 miles, nearly.

2. If the circumference of a circle is 326 rods, what is its diameter? *Ans.* 103.78 rods.

3. What is the circumference of a circle, whose diameter is 12 rods? *Ans.* 37.7 rods.

Solution by the sliding rule.—Place 3.14 on B under 1 on A, and under 12 found on the line A, will be found the answer on the line B, viz., 37.7 rods.

4. What is the diameter of a tree, whose circumference is $5\frac{1}{2}$ feet? *Ans.* 1.75 feet, nearly.

5. What is the diameter of a wheel, whose rim is 11 feet? *Ans.* 3.5014 feet.

6. What is the area of a circle, whose diameter is 20, and circumference 62.8318? *Ans.* 314.159.

7. What is the area of a circle, whose diameter is 226 links, and circumference 710? *Ans.* 40115 square links.

8. What is the area of a circular plantation, whose diameter is 640 links, and circumference 2010.6.

Ans. 3 acres, 0 roods, 34.56 rods.

9. What is the area of a circle whose diameter is 1, and circumference 3.1416? *Ans.* .7854.

Since the area of a circle, whose diameter is 1, is 0.7854

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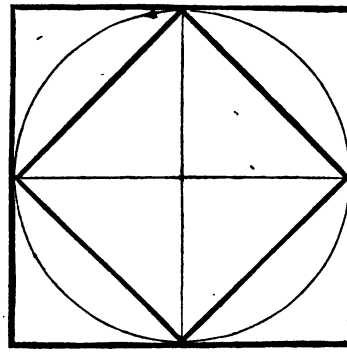
nearly, it follows, that the area of the circle is to the area of its circumscribing square as .7854 is to 1.



It is evident from the above figure, that, if the diameter of a circle is 1, the area of the circumscribing square will be 1, its side being equal to the diameter of the circle; but we have proved by example 9th, that the area of a circle, whose diameter is 1, is .7854; and consequently, the area of the circumscribing square is to the area of the circle as 1 to .7854. And it is likewise evident, from the above figure, that if the circumscribing square is 1, the inscribed square will be $\frac{1}{2}$, or .5; for in the circumscribing square there are eight equal triangles, and in the inscribed square there are but four. Therefore,

To find the area of a circle, its diameter being given:—

Multiply the square of the diameter by .7854: Or, *Multiply the square of the circumference by* 0.0796; or, by .0795775.

And, to find the area of the inscribed square:—

Divide the square of the diameter by 2.

The diameter of a circle being 1, the area of the inscribed square is .5, the square root of which is .707106: Therefore, the ratio of the diameter of a circle to the side of the inscribed square is as 1 to .707106781.

Therefore, to find the diameter of a circle in which you may inscribe a square of a given magnitude:—Divide the side of the

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square by .707106; or, the diameter of a circle being given, to find the side of the inscribed square:—*Multiply the diameter by* .707106; or, *Multiply the circumference by* .2251.

EXAMPLES.

1. What is the area of a circle, whose diameter is 450 links? *Ans.* 1 acre, 2 roods, 14.5 rods.

2. What is the area of a circle, whose diameter is 2? of one whose diameter is 3? of one whose diameter is 4? of one whose diameter is 9? and of one whose diameter is 25?

Solution by the Sliding Rule.

Place .7854 on the line C over 1 on the line D, and (because the areas of circles are as the squares of their diameters) over the diameter of any circle found on D, will be found its area on the line C. Having set the slider, as directed, over 2, you will find 3.14, or 3.1416, (the area being equal to the circumference of a circle whose diameter is 1;) and over 3 you will find 7.07; over 4, 12.57; over 9, 63.6; and over 25, 490.8.

3. How many inches square will a stick of timber be, if hewn or cut from a round log 15 inches in diameter? if cut from a log 20 inches in diameter? if cut from a log 30 inches in diameter? and of one cut from a log 40 inches in diameter?

Answers in order,—10.6 inches; 14.14 inches; 21.21 inches; and 28.28 inches square.

To perform this operation by the sliding rule:—Place .707 on the line B, under 1 on the line A; then under the diameter found on A will be found the side of the square stick on the line B: or, over the side of the square on B will be found the diameter of the circumscribing circle on the line A.

The circumference of a circle being given, to find its area:—Multiply the square of the circumference by 0.796.

4. What is the area of a circle, whose circumference is 20 feet 3 inches? *Ans.* 32.64 sq. feet.

To find the diameter of a circle, its area being given:—

Divide the given area by .7854, *and the quotient will be the*

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area of the circumscribing square, the square root of which will be the diameter.

5. What is the diameter of a circle, whose area is 144 square inches? *Ans.* 13.54 inches.

To solve this example by the sliding rule:—Place .7854 on C over 1 on D, then under any given area found on C will be found the diameter of a circle that will contain that area, on the line D.

6. What is the diameter of a circle, which will contain 1 acre, or 160 square rods? *Ans.* 14.28 rods.

7. What is the diameter of a circle, whose area is 1 square rod, or 272¼ square feet? *Ans.* 18.62 feet.

8. What is the diameter of a circle, whose area is 640 acres, or 1 square mile? *Ans.* 90.27 chains.

9. What is the diameter of a circle, whose area is 1728 inches? *Ans.* 47 inches, nearly.

10. What is the diameter of a circle, whose area is 231 square inches? *Ans.* 17.15 inches.

11. What is the diameter of a circle, whose area is 282 square inches? *Ans.* 18.95 inches.

12. What is the diameter of a circle, whose area is 2150.42 inches? *Ans.* 52.32 inches.

13. What is the diameter of a circle, whose area is 7276.5 square inches? *Ans.* 96.26 inches.

14. What is the diameter of a circle, whose area is 10,152 square inches? *Ans.* 113.63 inches.

The answers to the above examples are *gauge points* on the line D. Thus, to find the area of a circle in feet, the diameter being given in inches:—Place 1 on C over 13.54 on D, and over the diameter in inches found on D, will be found the area in feet on the line C. As 13.54 is a very important *gauge point,* (it being used to find the area of circles in feet, when the diameter is given in inches, and in computing the solid contents of round timber, and for several other purposes,) a point, or

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dot, should be made on the line D, a very little to the right of 13.5, for the purpose of enabling the practical mechanic to *set* the slider readily. This *dot*, or *point*, should be made by boring a very little distance into the rule with a needle, or some sharp-pointed instrument, and then the puncture should be filled with ink. A dot, or gauge point, should in like manner be made on the line D at 14.28, and at 17.15, and at 18.95, and at 13.387. See ¶ 58.

15. What is the area of a circle in feet, whose diameter is 25 inches? of one whose diameter is 30 inches? of one whose diameter is 70 inches? and of one whose diameter is 135.4 inches? *Answers*, 3.41; 4.9; 26.75; and 100 feet.

To find the area of a circle in acres, the diameter being given in rods:—Place 1 on C over 14.28 on D, and over the diameter in rods found on D, will be found the area in acres on the line C.

16. What is the area of a circle whose diameter is 35 rods? of one whose diameter is 70 rods? and of one whose diameter is 10 rods? The gauge point is 14.28.

Answers, 6 acres; 24 acres; 0.491 acres.

17. What is the diameter of a circle whose area is 10 acres? of one whose area is 20 acres? and of one whose area is 49 acres? *Answers*, 45.1 rods; 63.5 rods; 100 rods.

This example being the reverse of example 16th, under the area found on C, will be found the diameter on D.

To find the side of an equilateral triangle inscribed in a circle:—

From the square of the diameter of the circle, subtract the square of the radius, and the square root of the remainder will be the side of the triangle. Or, *from the square of the diameter of the circle subtract one-fourth of said square, and the square root of the remainder will be the side of the triangle.*

18. What is the side of a trigon inscribed in a circle whose diameter is 1? *Ans.* 0.866025403784.

Having found the side of a trigon drawn in a circle, whose

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diameter is 1, we may find the side of a trigon drawn in any other circle, by multiplying its diameter by the number which expresses the side of the trigon inscribed in the circle whose diameter is unity; or, if we divide the side of a trigon by said number, the quotient will be the diameter of the circumscribing circle.

The diameter of a circle being given, to find the side of an equal square:—

Multiply the diameter by the square root of .7854, viz. by .886227. Or, the circumference being given, to find the side of an equal square:—Multiply the circumference by .282094.

To find the side of an octagon drawn in a circle, its diameter being given:—

Multiply the diameter by 0.3826, and the product will be the side of the inscribed octagon: or, if we divide the side of an octagon by .3826, the quotient will be the diameter of the circumscribing circle.

To find the diameter of a circle in which you may inscribe a given square, or a given rectangle:—

Extract the square root of the sum of the squares of the length and breadth.

EXAMPLES.

1. What is the diameter of a circle, in which you may inscribe a rectangle 25 inches long and 15 broad?

Ans. 29.15 inches.

2. What must be the diameter of a round log, from which you may hew a stick of timber 12 inches by 20?

Ans. 23.324 inches.

3. What is the side of a square whose area is equal to that of a circle whose diameter is 20 rods?

Ans. 17.72454 rods.

4. What is the diameter of a circle, in which you can inscribe a trigon 30 inches on each side? *Ans.* 34.64105 inches.

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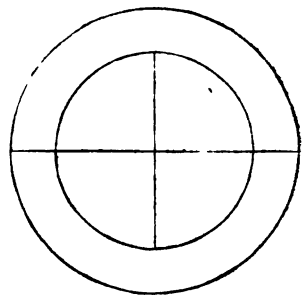
5. What is the diameter of a round log, from which you may form an octagonal prism, or column of eight equal sides, each side being 10 inches? *Ans.* 26.1367 inches.

¶ 23. CIRCULAR ANNULUS.



The *circular annulus* is the space included between the circumferences of two concentric circles. The difference between the areas of the two circles will manifestly be the area of the circular ring. Therefore, to find the area of a circular annulus:—

Multiply the sum of their diameters by their difference, and this product by .7854, *and the result will be the area.* Or,

Multiply the sum of their circumferences by their difference, and this product by .0796, *and the result will be the area.*

EXAMPLES.

1. What is the area of a gravelled walk 8.25 rods wide, extending round a circular fish-pond 10 rods in diameter?
Ans. 16.4934 square rods.

2. The circumferences of two concentric circles are 62.832 and 37.6992; required the area of the annulus contained between them. *Ans.* 201.0624.

3. If the internal diameter of one of Saturn's rings is 30,000 miles, and the external diameter is 50,000 miles; what is the area of one side of the ring, supposing it to be a flat surface?
Ans. 1,256,640,000 square miles.

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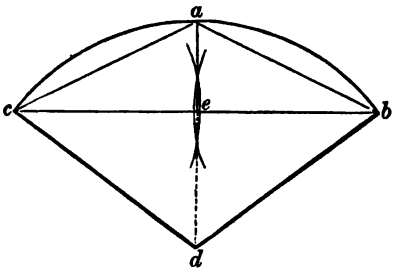
¶ 24. CIRCULAR SECTOR.



A sector is a part of a circle bounded by two radii and an arc. Its area is equal to that of a triangle whose base is the length of the arc, and whose perpendicular altitude is equal to the radius of the circle of which the sector is a part. Therefore, to find the area of a sector:—

Multiply the length of the arc by half the radius of the circle of which it is a part. Or, when the number of degrees in the arc is given, to find the area, say:—As 360 degrees are to the number of degrees in the arc, so is the area of the whole circle to the area of the sector.

When the number of degrees in the arc of the sector is given, to find the length of the arc, say:—As 360 degrees are to the number of degrees in the arc, so is the circumference of the circle to the length of the required arc. Or, multiply .0174533 by the number of degrees, and that product by the radius of the circle. Or,

From the sum of the chords of half the arc, (viz. *ac* and *ab,*) *subtract the chord of the whole arc,* (viz. *cb,*) *and add one-third of the remainder to the sum of the chords of half the arc, and the result will be the length of the required arc, nearly.*

The height of an arc (as *ae* in the above figure) is called the *versed sine;* and that part of the radius between the centre of the circle and the chord, (as *de* in the figure,) is called the *apothegm.*

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When the chord and versed sine of an arc are given, to find the diameter of the circle :—

Divide the square of half the chord by the versed sine, and to the quotient add the versed sine. Or, *Divide the square of the chord by four times the versed sine, and to the quotient add the versed sine.*

The apothegm and chord being given, to find the radius of the circle :—

To the square of the apothegm add the square of half the chord, and extract the square root of the sum of said squares.

The versed sine and chord of an arc being given, to find the chord of $\frac{1}{2}$ the arc :—

To the square of the versed sine add the square of half the chord, and extract the square root of the sum.

EXAMPLES.

1. What is the area of a sector, whose arc is 144.666, and the diameter of the circle, of which it is a part, 144 ?

Ans. 5207.8.

2. If the chord of half the arc is 126, and the chord of the whole arc 216, what is the length of the arc line ?

Ans. 264, nearly.

3. What is the length of an arc of 3 degrees, in a circle whose radius is 50 ? *Ans.* 2.618, nearly.

4. What is the area of a sector whose arc is 120 degrees, and the diameter of the circle 226 rods ?

Ans. 13371.66 square rods.

5. If the chord of an arc is 173.2, and the versed sine 50, what is the length of the arc ? *Ans.* 208.93, nearly.

6. If the height of an arc be 5.6, and the apothegm 8.4, what is the radius of the circle ? *Ans.* 14.

7. If the chord of an arc is 12, and the apothegm 10, what is the radius of the circle ? *Ans.* 11.6619.

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8. The versed sine of an arc is 10, and the chord of the arc 24; what is the radius of the circle? *Ans.* 12.2.

9. What is the diameter, when the versed sine is 1 and the cord 12? *Ans.* 37.

10. What is the radius of an arc, whose versed sine is 6, and the chord of half the arc 15? *Ans.* 18.75.

11. If the chord of an arc is 1200, and its versed sine 40, what is the diameter of the circle? *Ans.* 9040.

12. If the chord of an arc is 40, and the diameter of the circle 120, how many degrees does it contain?

Ans. 38° 57′.

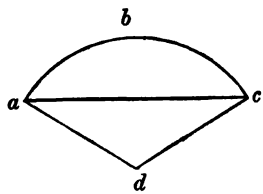
13. What is the number of degrees in an arc, whose versed sine is 12, and radius 56? *Ans.* 76° 25′.

14. What is the length of an arc of 45°, the diameter being 12? *Ans.* 4.712.

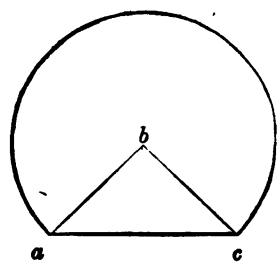
15. The length of a circular arc is 24, and the diameter of the circle 30; what is the area of the sector? *Ans.* 180.

¶ 25. CIRCULAR SEGMENTS.

LESS SEGMENT.



GREATER SEGMENT.



A circular segment is a part of a circle bounded by an arc and a chord.

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If the segment be *less* than a semicircle, to find its area:—

Find the area of the whole sector abcd, *of which it is a part, and from the area of the sector subtract the area of the triangle* acd, *included between the radii and chord, and the remainder will be the area of the less segment.*

To find the area of a segment greater than a semicircle:—

Find the area of the whole sector, to which add the area of the triangle abc, *included between the radii and the chord.*

To find the arc line in the greater segment:—Divide the arc into two equal parts, and then proceed with each half as directed under the sector.

EXAMPLES.

1. What is the area of a segment less than a semicircle, whose chord is 172, the chord of half the arc 104, and the versed sine, or height of the arc, 58.48? *Ans.* 7248.25.

By the rules under the sector, we find the arc line, *abc*, to be 220, the diameter of the circle 184.95, and the area of the sector 10172.25; and by the rules under the triangle, ¶ 19, we find the altitude of the triangle to be 33.995, and its area 292, which, subtracted from the area of the sector, leaves 7248.25 for the area of the segment.

2. In a greater segment, the chord is 136, the chord of $\frac{1}{3}$ the arc is 146, the chord of $\frac{1}{4}$ of the arc is 86, and the radius of the circle is 80; what is the area of the segment?

Ans. 17309.28.

We find the arc of half the segment to be 180.666, and the whole arc 361.332, and the area of the sector 14453.28, the altitude of the triangle 42 nearly, and its area 2856, which, being added to the area of the sector, gives 17309.28 for the area of the circular segment.

3. What is the area of a circular segment less than a semicircle, its chord being 40, and height 4? *Ans.* 107.515.

4. The chord of a segment is 20, and its height is 5; what is its area? *Ans.* 69.81.

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5. What is the area of a segment, whose chord is 16, and the diameter of the circle 20? *Ans.* 44.73.

6. What is the area of a segment, whose arc is a quadrant, or 90 degrees, and the diameter 12 feet?

Ans. 10.274 square feet.

The chord in this example will evidently be equal to the side of a square inscribed in the circle; and consequently, to find the chord multiply the diameter by .7071065; and the height of the triangle will be half the length of the chord, and the length of the arc will be one-fourth the circumference of the circle.

The area may also be found by means of a table containing the areas of segments of a circle, whose diameter is 1, and whose heights, or versed sines, are all the numbers between 0 and 1 carried to any number of decimal places.

To find the area of a circular segment by this method:—

Divide the versed sine by the diameter of the circle, (and the quotient is the height of the similar segment when the diameter is 1;) *take the tabular area corresponding to the quotient, and multiply it by the square of the diameter, and the product is the area of the given segment.*

EXAMPLES.

1. The diameter of a circle is 30 feet, and the versed sine of the segment is 6 feet; what is its area? *Ans.* 100.64 feet.

Solution.— $\frac{6}{30} = .2$, the height of the similar segment whose diameter is 1. Look for .2 in the column of heights in the table, and against it you will find .111824; and $.111824 \times 30 \times 30 = 100.6416$.

2. The diameter of a circle is 100 feet, and the height of a segment is 6.5 feet; what is its area? *Ans.* 216.69 feet.

Solution.— $\frac{6.5}{100} = .065$, and the tabular area for this height is .021669, which multiplied by the square of 100=216.69. The other exercises given above, may be performed in the same manner to exemplify this rule. When the area of a greater segment is required, find the tabular area of the less segment, and subtract it from .7854, and the remainder will be the tabular area of the greater segment.

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A TABLE

OF THE AREAS OF THE SEGMENTS OF A CIRCLE,

The Diameter of which is Unity, and supposed to be divided into 100 equal parts.

Height.	Area.	Height.	Area.	Height.	Area.	Height.	Area.
·001	·000042	·043	·011734	·085	·032186	·127	·057991
·002	·000119	·044	·012142	·086	·032746	·128	·058658
·003	·000219	·045	·012555	·087	·033308	·129	·059328
·004	·000337	·046	·012971	·088	·033873	·130	·059999
·005	·000471	·047	·013393	·089	·034441	·131	·060673
·006	·000619	·048	·013818	·090	·035012	·132	·061349
·007	·000779	·049	·014248	·091	·035586	·133	·062027
·008	·000952	·050	·014681	·092	·036162	·134	·062707
·009	·001135	·051	·015119	·093	·036742	·135	·063389
·010	·001329	·052	·015561	·094	·037324	·136	·064074
·011	·001533	·053	·016008	·095	·037909	·137	·064761
·012	·001746	·054	·016458	·096	·038497	·138	·065449
·013	·001969	·055	·016912	·097	·039087	·139	·066140
·014	·002199	·056	·017369	·098	·039681	·140	·066833
·015	·002438	·057	·017831	·099	·040277	·141	·067528
·016	·002685	·058	·018297	·100	·040875	·142	·068225
·017	·002940	·059	·018766	·101	·041477	·143	·068924
·018	·003202	·060	·019239	·102	·042081	·144	·069626
·019	·003472	·061	·019716	·103	·042687	·145	·070329
·020	·003749	·062	·020197	·104	·043296	·146	·071034
·021	·004032	·063	·020681	·105	·043908	·147	·071741
·022	·004322	·064	·021168	·106	·044523	·148	·072450
·023	·004619	·065	·021669	·107	·045140	·149	·073162
·024	·004922	·066	·022155	·108	·045759	·150	·073875
·025	·005231	·067	·022653	·109	·046381	·151	·074590
·026	·005546	·068	·023155	·110	·047006	·152	·075307
·027	·005867	·069	·023660	·111	·047633	·153	·076026
·028	·006194	·070	·024168	·112	·048262	·154	·076747
·029	·006527	·071	·024680	·113	·048894	·155	·077470
·030	·006866	·072	·025196	·114	·049529	·156	·078194
·031	·007209	·073	·025714	·115	·050165	·157	·078921
·032	·007559	·074	·026236	·116	·050805	·158	·079650
·033	·007913	·075	·026761	·117	·051446	·159	·080380
·034	·008273	·076	·027290	·118	·052090	·160	·081112
·035	·008638	·077	·027821	·119	·052737	·161	·081847
·036	·009008	·078	·028356	·120	·053385	·162	·082582
·037	·009383	·079	·028894	·121	·054037	·163	·083320
·038	·009763	·080	·029435	·122	·054690	·164	·084060
·039	·010148	·081	·029979	·123	·055346	·165	·084801
·040	·010538	·082	·030526	·124	·056004	·166	·085545
·041	·010932	·083	·031077	·125	·056664	·167	·086290
·042	·011331	·084	·031630	·126	·057327	·168	·087037

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Height.	Area.	Height.	Area.	Height.	Area.	Height.	Area.
·169	·087785	·219	·127286	·269	·170202	·319	·215734
·170	·088536	·220	·128114	·270	·171090	·320	·216666
·171	·089288	·221	·128943	·271	·171978	·321	·217600
·172	·090042	·222	·129773	·272	·172868	·322	·218534
·173	·090797	·223	·130605	·273	·173758	·323	·219469
·174	·091555	·224	·131438	·274	·174650	·324	·220404
·175	·092314	·225	·132273	·275	·175542	·325	·221341
·176	·093074	·226	·133109	·276	·176436	·326	·222278
·177	·093837	·227	·133946	·277	·177330	·327	·223216
·178	·094601	·228	·134784	·278	·178226	·328	·224154
·179	·095367	·229	·135624	·279	·179122	·329	·225094
·180	·096135	·230	·136465	·280	·180020	·330	·226034
·181	·096904	·231	·137307	·281	·180918	·331	·226974
·182	·097675	·232	·138151	·282	·181818	·332	·227916
·183	·098447	·233	·138996	·283	·182718	·333	·228858
·184	·099221	·234	·139842	·284	·183619	·334	·229801
·185	·099997	·235	·140689	·285	·184522	·335	·230745
·186	·100774	·236	·141538	·286	·185425	·336	·231689
·187	·101553	·237	·142388	·287	·186329	·337	·232634
·188	·102334	·238	·143239	·288	·187235	·338	·233580
·189	·103116	·239	·144091	·289	·188141	·339	·234526
·190	·103900	·240	·144945	·290	·189048	·340	·235473
·191	·104686	·241	·145800	·291	·189956	·341	·236421
·192	·105472	·242	·146655	·292	·190865	·342	·237369
·193	·106261	·243	·147513	·293	·191774	·343	·238319
·194	·107051	·244	·148371	·294	·192685	·344	·239268
·195	·107843	·245	·149231	·295	·193597	·345	·240219
·196	·108636	·246	·150091	·296	·194509	·346	·241170
·197	·109431	·247	·150953	·297	·195423	·347	·242122
·198	·110227	·248	·151816	·298	·196337	·348	·243074
·199	·111025	·249	·152681	·299	·197252	·349	·244027
·200	·111824	·250	·153546	·300	·198168	·350	·244980
·201	·112625	·251	·154413	·301	·199085	·351	·245935
·202	·113427	·252	·155281	·302	·200003	·352	·246890
·203	·114231	·253	·156149	·303	·200922	·353	·247845
·204	·115036	·254	·157019	·304	·201841	·354	·248801
·205	·115842	·255	·157891	·305	·202762	·355	·249758
·206	·116651	·256	·158763	·306	·203683	·356	·250715
·207	·117460	·257	·159636	·307	·204605	·357	·251673
·208	·118271	·258	·160511	·308	·205528	·358	·252632
·209	·119083	·259	·161386	·309	·206452	·359	·253591
·210	·119898	·260	·162263	·310	·207376	·360	·254551
·211	·120713	·261	·163141	·311	·208302	·361	·255511
·212	·121530	·262	·164020	·312	·209228	·362	·256472
·213	·122348	·263	·164900	·313	·210155	·363	·257433
·214	·123167	·264	·165781	·314	·211083	·364	·258395
·215	·123988	·265	·166663	·315	·212011	·365	·259358
·216	·124811	·266	·167546	·316	·212941	·366	·260321
·217	·125634	·267	·168431	·317	·213871	·367	·261285
·218	·126459	·268	·169316	·318	·214802	·368	·262249

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Height.	Area.	Height.	Area.	Height.	Area.	Height.	Area.
·369	·263214	·402	·295330	·435	·327883	·468	·360721
·370	·264179	·403	·296311	·436	·328874	·469	·361719
·371	·265145	·404	·297292	·437	·329866	·470	·362717
·372	·266111	·405	·298274	·438	·330858	·471	·363715
·373	·267078	·406	·299256	·439	·331851	·472	·364714
·374	·268046	·407	·300238	·440	·332843	·473	·365712
·375	·269014	·408	·301221	·441	·333836	·474	·366711
·376	·269982	·409	·302204	·442	·334829	·475	·367710
·377	·270951	·410	·303187	·443	·335823	·476	·368708
·378	·271921	·411	·304171	·444	·336816	·477	·369707
·379	·272891	·412	·305156	·445	·337810	·478	·370706
·380	·273861	·413	·306140	·446	·338804	·479	·371705
·381	·274832	·414	·307125	·447	·339799	·480	·372704
·382	·275804	·415	·308110	·448	·340793	·481	·373704
·383	·276776	·416	·309096	·449	·341788	·482	·374703
·384	·277748	·417	·310082	·450	·342783	·483	·375702
·385	·278721	·418	·311068	·451	·343778	·484	·376702
·386	·279695	·419	·312055	·452	·344773	·485	·377701
·387	·280669	·420	·313042	·453	·345768	·486	·378701
·388	·281643	·421	·314029	·454	·346764	·487	·379701
·389	·282618	·422	·315017	·455	·347760	·488	·380700
·390	·283593	·423	·316005	·456	·348756	·489	·381700
·391	·284569	·424	·316993	·457	·349752	·490	·382700
·392	·285545	·425	·317981	·458	·350749	·491	·383700
·393	·286521	·426	·318970	·459	·351745	·492	·384699
·394	·287499	·427	·319959	·460	·352742	·493	·385699
·395	·288476	·428	·320949	·461	·353739	·494	·386699
·396	·289454	·429	·321938	·462	·354736	·495	·387699
·397	·290432	·430	·322928	·463	·355733	·496	·388699
·398	·291411	·431	·323919	·464	·356730	·497	·389699
·399	·292390	·432	·324909	·465	·357728	·498	·390699
·400	·293370	·433	·325900	·466	·358725	·499	·391699
·401	·294350	·434	·326891	·467	·359723	·500	·392699

The areas of the circular zone, the lune, and the elliptic segments, may be readily found by means of this table; it will also be found particularly useful in calculating the solidities of the cylindric and conic ungulas, and the solidities of vaulted arches, &c.

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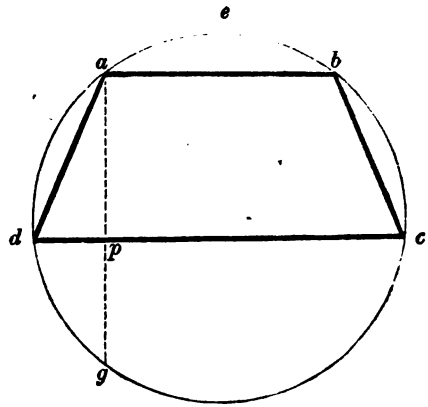
¶ 26. CIRCULAR ZONE.

A circular zone is a figure contained between two parallel chords and the intercepted arcs.



To find the area of a circular zone:—

Find the area of the segment *daebc*, from which subtract the segment *aeb*, and the remainder will be the area of the zone. Or, find the area of the trapezoid *dabc*, to which add twice the area of the segment *bc*, or *ad*, and the sum will be the area of the zone.

EXAMPLES.

1. What is the area of a circular zone, the chords which bound it on two sides being 20 and 30, and the perpendicular *ap*, between them, 12, and the chord *bc* 16, and the height of its arc 2? *Ans.* 334.72.

2. What is the area of a circular zone, the parallel chords of which are 90 and 50, and the distance between them 20? *Ans.* 1474.

To find the diameter of the circle in example second:—Mul-

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tiply the sum of the chords, *ab* and *cd*, by their difference, and divide the product by four times the distance between them, or four times *ap*, and to the quotient add *ap*, or 20, and the sum is *ag*; then add the square of *ag* to the square of the chord *ab*, and extract the square root of the sum of the squares, and said root will be the diameter, viz. 102.956; and half the difference between *ab* and *dc*, viz. 20, will be *dp*; and the square of radius, *minus* the square of half the chord *ad*, will give the square of the apothegm falling on the centre of the chord *ad*, the root of which will be the apothegm. The chord *ad* is 28.2825, and the apothegm is 49.4975, which subtracted from radius, leaves the height or versed sine of the arc *ad*, or *bc* = 1.9805; and the length of the arc *ad* = 28.64, nearly; the area of the segment *ad* = 37.1649, which, doubled and added to the area of the trapezoid, *dabc*, gives the area of the zone.

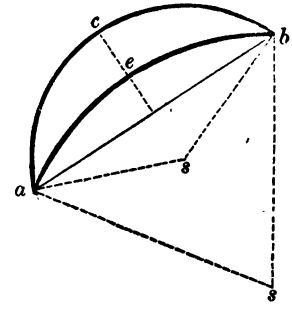
3. The chords of a circular zone are 30 and 48, and the distance between them 13; required its area. *Ans.* 534.188.

¶ 27. LUNE, OR CRESCENT.

A lune is the area included between the arcs of two eccentric circles which have a common chord.

To find the area of a lune:—

Find the areas of the two segments, *acb* and *aeb*, which stand on the same chord, and their difference is the area of the lune.



EXAMPLES.

1. In a lunar surface the chord *ab* is 88, the height of the segment *aeb* is 20, and the height of the segment *acb*, above the chord *ab*, is 40; find the area of the lune. *Ans.* 1452.

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The radius of the circle, the height of whose arc is 20, I find to be 58.4, and the chord of half the arc of this segment is 48.332, and the length of the arc 99.556, and the area of the segment 1,218, nearly. The radius of the circle, the height of whose segment is 40, I find to be 44.2, and the length of the chord of half the arc 59.464, and the length of the arc 129.237, and its area 2,670, which area, *minus* the area of the other segment, leaves the area of the crescent.

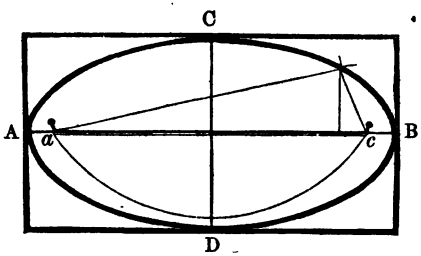
2. The chord is 48, and the heights, or versed sines, of the two segments are 7 and 18; what is the area?

Ans. 405.87.

¶ 28. MENSURATION OF THE CONIC SECTIONS.

The conic sections are three curves, the *parabola, ellipse,* and *hyperbola.*

ELLIPSE.



An ellipse is produced by cutting a cone by a plane passing obliquely through its opposite sides.

To draw an ellipse:—Draw the *major* axis, AB, of any required length, and through its centre and at right angles to it, draw the *minor* axis, CD, of any required length; then take half of the major axis between the dividers, and having placed one foot on the extremity of the minor axis at C, strike an arc cutting the major axis at two points, *a* and *c*, and these two

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points will be the *foci* of the ellipse. Drive a pin or nail into each of these points, and put a small thread or cord round them, and tie it so that when extended it will just reach the end of the major or minor axis; then with a style or pencil within the thread, extend the string, and describe the ellipse, keeping the thread always stretched, and suffering it to revolve freely about the two pins.

The ellipse bears the same ratio to its least circumscribing rectangle, that the circle bears to its least circumscribing square. Therefore, to find the area of an ellipse:—

Multiply the major by the minor axis, and that product by .7854.

To find the circumference of an ellipse:—

Add together the squares of the major and minor axes, and extract the square root of this sum, and to double this root add one-third of the minor axis. Or, *Find the square root of half the sum of the squares of the two diameters, and multiply this root by* 3.1416, and the product will be the circumference, nearly; or, the mean between the results found by these two rules, will commonly be a still nearer approximation.

An *ordinate* to either axis, is a line falling perpendicularly upon it from any point in the curve; and this line produced to meet the curve on the other side of the axis, is called a *double ordinate;* and each of the segments, or parts, into which the ordinate divides the axis, is called an *abscis*, or *abscissa.*

Thus, *eg* is an ordinate, and A*g* and B*g* are the abscisses. The *parameter* is the same as the double ordinate drawn through the focus; and this line is always a *third proportional* to the axis on which it falls and the other axis.

When the two axes and an abscis are given, to find the ordinate:—

Say: As the square of the major axis is to that of the minor, so is the product of the two abscisses into each other, to the square of the required ordinate.

When the two axes are given and an ordinate, to find the abscisses:—

Say: As the square of the minor axis is to the square of the

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major axis, so is the product of the sum and difference of the semi-minor axis and the ordinate, to the distance of the ordinate from the centre of the ellipse; which distance being added to, and subtracted from the semi-major axis, will give the greater and less abscisses.

When the minor axis, an ordinate, and an abscis, are given, to find the major axis:—

Find the square root of the difference of the squares of the semi-minor axis and the ordinate, and, according as the less or greater abscis is given, add this root to or subtract it from the semi-minor axis; then, as the square of the ordinate is to the product of the abscis and minor axis, so is the sum or difference, found above, to the major axis.

When the major axis, an ordinate, and one of the abscisses, are given, to find the minor axis:—

Find the other abscis; then the product of the two abscisses is to the square of the ordinate, as the square of the major axis to that of the minor axis.

When the two axes are given to find the *eccentricity* of the ellipse, that is, the distance of the foci from the minor axis:—

Extract the square root of the difference of the squares of the two axes, and half of said root will be the eccentricity.

EXAMPLES.

1. What is the area of an ellipse, whose major or transverse axis is 36 feet, and its minor or conjugate axis is 28 feet?

Ans. 791.68 square feet.

To solve this and all similar examples by the sliding rule:—Place the major axis found on C over the same on D; then find the minor axis on C, and under it you will find the diameter of a circle on D, whose area is equal to that of the ellipse, viz. 31.74; then place .7854 found on C, over 1 on D, and over the diameter of the equal circle, viz. 31.74 found on D, will be found the area on C.

2. What is the area of an ellipse, whose major axis is 88 rods, and its minor 72? *Ans.* 31.1 acres, nearly.

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To solve this example by the sliding rule:—Find the diameter of the equal circle, then place 1 on C over 14.28 on D, (the gauge point for acres in a circle,) and over the diameter of the equal circle found on D, will be found the area on C.

3. What is the area of an ellipse whose axes are 70 and 50 rods? *Ans.* 17 acres, 29 rods, nearly.

4. What is the circumference of an ellipse whose axes are 40 and 60? By the first rule we find the circumference to be 157.555, and by the second, 160,22, the mean between which is 158.8875, which is the perimeter of the ellipse, nearly. When the ellipse does not differ much from a circle, the *second rule* will be found the most accurate; but when the eccentricity is great, the *first rule* should only be used.

5. What is the circumference of an ellipse whose axes are 40 and 10? *Ans.* 85.8.

6. The axes of an ellipse are 8 and 6; what is the length of the curve? *Ans.* 22.2, nearly.

7. What is the eccentricity of an ellipse whose axes are 30 and 50? *Ans.* 20.

8. The major and minor axes of an ellipse are 60 and 20, and one absciss is 12; it is required to find the ordinate. *Ans.* 8.

9. The axes are 45 and 15, and one absciss is 9; what is the ordinate? *Ans.* 6.

10. The axes are 45 and 15, and the ordinate 6; what are the abscisses? *Ans.* 36 and 9.

11. The axes are 30 and 50; what is the parameter, or double ordinate passing through the focus?

50 : 30 : : 30 is to the parameter. *Ans.* 18.

12. The minor axis is 15, an ordinate 6, and the less absciss 9; what is the major axis? *Ans.* 45.

13. The minor axis is 50, an ordinate 20, and the less absciss 14; the major axis is required. *Ans.* 70.

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14. When the major axis is 15, an ordinate 2, and an abscissa 3, what is the minor axis? *Ans.* 5.

15. If the major axis is 70, and an ordinate 20, and one of the abscisses 14, what is the minor axis? *Ans.* 50.

The ellipse is the curve in which the planets perform their revolutions round the sun, and its properties enter into almost every investigation connected with physical astronomy.

It is a singular and curious property of the ellipse, that if a moving or generating circle roll along the concave circumference of another circle in the same plane, the radius of the generating circle being half that of the other, any given point in the circumference or plane of the moving circle will describe an ellipse. An ellipse is most conveniently drawn by an instrument called the trammel, usually possessed by master builders.

ELLIPTICAL SEGMENTS.

An elliptical segment bears the same proportion to the corresponding circular segment, that the whole ellipse does to the whole circle described on the axis of which the height of the segment is a part. *Hence, if either axis of an ellipse be made the diameter of a circle, and a line perpendicular to this axis cuts off a segment from the ellipse and from the circle;* THE DIAMETER OF THE CIRCLE WILL BE TO THE OTHER AXIS OF THE ELLIPSE, AS THE CIRCULAR SEGMENT IS TO THE ELLIPTIC SEGMENT.

EXAMPLES.

1. What is the area of an elliptic segment, cut off by an ordinate or chord perpendicular to the major axis, the major axis being 415 feet, the minor 332 feet, and the height, or abscissa of the segment, 96 feet? *Ans.* 18876.8 square feet.

In this example, the radius of the corresponding circular segment will be $\frac{4}{2}\frac{1}{5}$, or 207.5, and its versed sine 96 feet; and consequently, the apothegm, or height of the triangle formed by the radii of the circular sector and the segment's base, will be 111.5; and twice the square root of the difference

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of the height of the triangle and the radius will be the base of the circular segment, viz. 350, and the square root of the sum of the squares of the versed sine and half the base will be the chord of half the arc, viz. 199.6, and the arc line will = 415.6, and the area of the circular segment will = 23596. Then, as 415 : 332 : : 23596 to the area of the elliptic segment.

Or, to find the segment of an ellipse:—

Divide the height of the segment by the axis of the ellipse, which is perpendicular to the base of the segment; find the tabular area corresponding to this quotient, in the table of circular segments, and multiply it by the product (or rectangle) of the two axes of the ellipse, and the result will be the area.

2. What is the area of an elliptic segment, whose base is parallel to the minor axis, the height of the segment being 10 feet, and the axes of the ellipse 35 and 25? *Ans.* 162.021.

Solution.—$\frac{10}{35}$ = .2857, and the tabular area corresponding to this number is .185167, and $.185167 \times 35 \times 25 = 162.02$.

3. Find the area of an elliptic segment, whose base is parallel to the major axis, its height being 2, and the diameters 14 and 10. *Ans.* 15.6553.

¶ 29. PARABOLA.

A *parabola* is a figure formed by cutting a section from a cone by a plane parallel to one of its sides.

The nature of the curve is such that every point in it is equally distant from a certain point F, and a given straight line GH. The given point F is called the *focus* of the parabola, and the given right line GH, is called the *directrix.*

The point D is called the *vertex* of the parabola; and the line DC, which is perpendicular to the directrix, is called the *axis*, or *principal diameter.*

An *ordinate* is a perpendicular from any point in the curve on the axis, as SN: and when the ordinate is produced to

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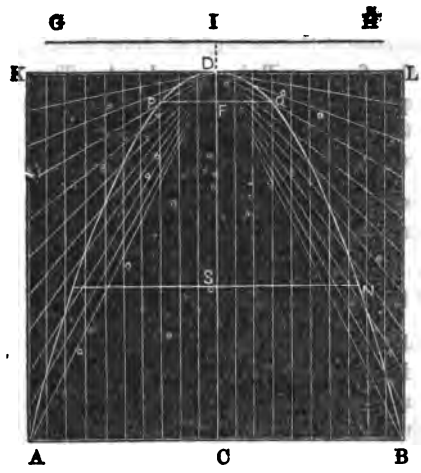

meet the curve on the other side of the axis, it is called a *double* ordinate; and the portion of the axis intercepted between the ordinate and the vertex of the curve, is called the *abscis*s, as DS or DC. The double ordinate PQ, passing through the focus, is called the *parameter*, or *latus rectum*; this line is always equal to four times the abscis DF, (or to four times its distance from the vertex,) and its distance from the vertex is always half its distance from the directrix: consequently, DF and DI are equal.

To construct a parabola:—

Draw the base AB of any required length, and on the extremities of the base, or *double ordinate;* erect perpendiculars, and make BL and AK equal to the abscis of the parabola, and complete the rectangle by joining KL. Then divide the lines AB and KL into an even number of equal parts, and through each of the divisions, or points, draw lines parallel to the sides LB or AK; then divide the other two sides of the rectangle into half as many equal parts as the former, and from each of the divisions in the sides LB and AK, draw lines meeting at the centre of the line KL, and the vertex of the parabola will be at this point, and its curve will pass through the points

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where the parallel lines are cut by the converging: consequently, commence at the vertex, and draw lines through these points, and the figure will be completed. The greater the number of equal parts into which the sides of the rectangle are divided, the greater will be the number of points found in the curve, and the more accurately it may be drawn.

When an ordinate of a parabola and its absciss are given, to find the parameter:—

Divide the square of the ordinate by the absciss, and the quotient will be the parameter.

Any two abscisses are to each other as the squares of their respective ordinates; and the absciss of any given ordinate, is to that of any required ordinate, as the square of the given ordinate to the square of the required ordinate; and the square of any given ordinate is to its absciss, as the square of any other given ordinate is to its required absciss.

The parabola is exactly two-thirds of its least circumscribing rectangle. Therefore, to find its area:—

Multiply its base, or double ordinate, by two-thirds of its altitude or absciss.

Or, Multiply any double ordinate by two-thirds of its absciss, and the product will be the area of that part of the parabola between the ordinate and vertex.

To find the area of a zone of a parabola:—

From the area of the whole parabola subtract the area of the segment or part above the less ordinate.

Or, To one of the parallel sides add the quotient obtained by dividing the square of the other by the sum of the two parallel sides; and multiply this sum by $\frac{2}{3}$ of the altitude of the zone.

To find the length of the parabolic curve:—

To the square of the ordinate, or half the base, add four-thirds of the square of the abscis, and the square root of the sum, multiplied by two, will be the length of the curve, nearly.

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EXAMPLES.

1. If the ordinate of a parabola is 6, and its absciss 15, what is its parameter? *Ans.* 2.4.

2. The ordinate of a parabola is 20, and its absciss 36; find its parameter. *Ans.* $11\frac{1}{9}$.

3. The abscisses of a parabola are 18 and 32, and the ordinate of the former is 12; required the ordinate of the latter. *Ans.* 16.

4. Two ordinates are 18 and 24, and the abscis of the former is 18; find that of the latter. *Ans.* 32.

5. What is the area of a parabola whose base is 25 and altitude 18? *Ans.* 300.

6. Find the area of a parabola whose double ordinate is 36 and abscis 45. *Ans.* 1080.

7. What is the area of a parabola whose base is 26 inches, and abscis 9 feet? *Ans.* 13 feet.

8. What is the area of a parabola whose base is 16 rods, and altitude 10? *Ans.* 106.666 rods.

9. What is the length of a parabolic curve, the double ordinate being 18 and its altitude 24? *Ans.* 58.275.

10. The abscis and ordinate are 10 and 8; what is the length of the curve? *Ans.* 28.09.

11. What is the area of a parabolic zone, the parallel ordinates being 18 and 30, and the altitude 9? *Ans.* 220.5.

12. What is the area of a parabolic zone, whose parallel ordinates are 6 and 10, and height 6? *Ans.* 49.

13. The parallel sides of a parabolic zone are 10 and 15, and the distance between them 15; required its area. *Ans.* 190.

All parabolas are similar figures; and hence two parabolas which have the same parameter and abscis are equal to each other. If a parabola and an ellipse have the same vertex and focus, then the major axis of the ellipse is to the distance of

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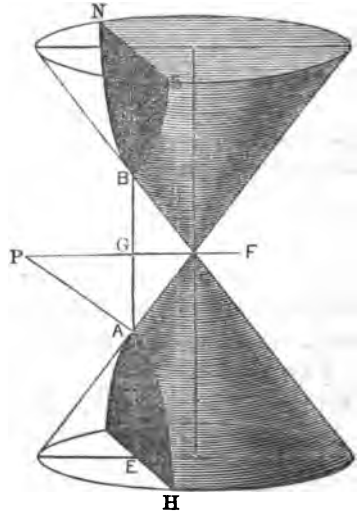
the focus from the other vertex, as the parameter of the parabola to the parameter of the ellipse. If a parabola and ellipse have the same focus and parameter, the distances of the vertexes will be in the same ratio. Parallel rays of light falling on the concave surface of a parabola, or a concave surface generated by the revolution of a parabola about its axis, are all collected in the focus, which is called the *burning point*. All projectiles moving in a vacuum describe a parabolic curve; and heavy bodies, as iron or leaden balls, moving through the air with a velocity not exceeding 1142 feet in a second, describe a parabolic curve very nearly; and on the properties of this curve is founded the theory of gunnery. See ¶ 73.

¶ 80. HYPERBOLA.



An *hyperbola* is a plane figure formed by cutting a section from a cone by a plane parallel to its axis, or to any plane within the cone which passes through the cone's vertex.

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The curve of the hyperbola is such, that the difference between the distances of any point in it from two given points is always equal to a given right line.

If the vertexes of two cones meet each other so that their axes form one continuous straight line, and the plane of the hyperbola cut from one of the cones be continued, it will cut the other cone, and form what is called the *opposite hyperbola*, *equal* and *similar* to the former: and the distance between the vertexes of the two hyperbolas is called the *major axis*, or *transverse* diameter. Thus AB is the major axis. If the distance between a certain point within the hyperbola, called the *focus*, and any point in the curve, be subtracted from the distance of said point in the curve from the focus of the opposite hyperbola, the remainder will always be equal to a *given quantity*, that is, to the *major axis:* and the distance of either focus from the centre of the major axis is called the *eccentricity*. The line passing through the centre perpendicular to the major axis, and having the distance of its extremities from those of this axis equal to the eccentricity, is called the *minor axis*, or *conjugate diameter*. An *ordinate* to the major axis, a *double ordinate*, and an *absciss*, are to be understood to mean the same as the corresponding lines in the parabola. Thus AB in the figure is the major axis, GC or GR is the eccentricity, C and R being the foci. The line PF is the minor axis, the distances of P and F from A and B being equal to GR, or GC, or AP. EH is an ordinate, and this line produced to meet the curve on the opposite side is called a double ordinate; and AC or AE is an absciss, and BC or BE is a greater absciss. A third proportional to the major and minor axes is called the *parameter* of the major axis. Thus a third proportional to AB and PF is the parameter of AB, and is equal to the double ordinate passing through the focus at C or R.

Properties of the hyperbola.

1. As the semi-major axis, AG, is to the distance of the focus from the centre, GC, so is the distance of any ordinate from the centre, to half the sum of the distances of the point in the

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curve, on which the said ordinate falls, from the foci of the two hyperbolas.

2. As the square of the major axis, AB^2, is to the square of the minor axis, PF^2, so is the product of the two abscisses (that is, BE into AE) to the square of the ordinate, EH.

3. As the transverse axis is to the parameter, so is the rectangle of the distances of any ordinate from the vertexes of the two hyperbolas, to the square of that ordinate. Hence, when the transverse, or major axis, and the parameter and abscis are given, the length of any number of ordinates may be found; and if we divide the abscis into several equal parts, and calculate the length of the ordinates to those parts, and divide double the sum of the ordinates by their number, and multiply the quotient by the abscis, we shall obtain the area nearly.

4. If any line be drawn through the focus of the hyperbola, meeting the curve on opposite sides, the rectangle of the two segments between the focus and the curve will equal the rectangle, or product, of half of that line into half of the parameter.

5. In an equilateral hyperbola, (that is, one having the two axes equal,) the square of the semi-major axis is equal to the product obtained by multiplying the distance of the focus from the vertex by the distance of the vertex from the focus of the opposite hyperbola. That is, $AG \times AG = AC \times AR$.

6. In an equilateral hyperbola, the product of the distances of any ordinate from the vertexes of the two hyperbolas, is equal to the square of that ordinate.

7. The areas of two hyperbolas having the same major axis and the same abscis, are to each other as their minor axes.

8. As the square of the minor axis is to the square of the major axis, so is the sum of the squares of the semi-minor axis and an ordinate, to the square of the distance between that ordinate and the centre. The sum of this distance and the semi-minor axis will give the greater abscis; and their difference, the less.

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9. The product of the abscisses is to the square of the ordinate, as the square of the major axis to that of the minor axis.

10. The minor axis, an ordinate, and two abscisses being given, to find the major axis:—

Find the square root of the sum of the squares of the semi-minor axis and the ordinate; then, according as the less or greater abscis is given, find the sum or difference of this root and the semi-minor axis; then say:—As the square of the ordinate is to the product of the abscis and minor axis, so is the sum or difference found above, to the major axis.

11. To find the length of an arc of an hyperbola, reckoning from the vertex of the curve:—

To 15 *times the major axis add* 21 *times the less abscis, and multiply the sum by the square root of the minor axis; add this product to* 19 *times the product of the square of the major axis by the abscis, and add it also to* 9 *times the same product; divide the former sum by the latter, and multiply the quotient by the ordinate, and the product will be the length of the arc very nearly.*

12. To find the area of an hyperbola, the axes and an ordinate being given:—

To 7 *times the major axis add* 5 *times the less abscis; multiply the sum by* 7 *times the abscis, and the square root of this product by* 3; *to this product add* 4 *times the square root of the product of the major axis and abscis; multiply this sum by* 16 *times the product of the minor axis and abscis; divide this product by* 300 *times the major axis, and the quotient will be the required area nearly.*

EXAMPLES.

1. If the semi-major axis of an hyperbola be 10, and the distance of the focus from the centre of the major axis be 12, and the distance of an ordinate from the centre be 15, what is the sum of the distances of that point in the curve on which the ordinate falls, from the two foci? *Ans.* 36.

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2. The major axis of an hyperbola is 15, the minor axis 9, and the less abscis 5; what is the length of the ordinate?

Ans. 6.

The less abscis being 5, the greater will be 15+5, or 20; then by property 2d:—225 : 81 :: 100 to the square of the ordinate; consequently, the square root of 36, viz. 6, is the required ordinate.

3. The minor and major axes are 48 and 42, and the less abscis 16; what is the ordinate? *Ans.* 28.

4. The major axis is 25, the minor 15, and the less abscis $8\frac{1}{3}$; what is the ordinate? *Ans.* 10.

5. The major axis is 30, the parameter 6, and the distances of an ordinate from the two vertexes 12 and 42; what is the length of the ordinate? *Ans.* 4.1, nearly.

6. If the segments of a line passing through the focus of an hyperbola, and meeting the curve on opposite sides, are 20 and 8, what is the length of the parameter? *Ans.* $22\frac{2}{7}$.

7. In an equilateral hyperbola, the semi-major axis is 10; required the distance of the focus of the hyperbola from its vertex. *Ans.* 4.1421.

The solution of this question is derived from the fifth property of the hyperbola. Thus:—Extract the square root of twice the square of the semi-major axis, and subtract the semi-major axis from this root, and the remainder will be the required distance.

8. In an equilateral hyperbola, the distances of an ordinate from the vertexes are 25 and 49; required the ordinate.

Ans. 35.

9. Two hyperbolas have the same major axes, and their abscisses are the same; the minor axis of one of them is 20, and the minor axis and area of the other are 15 and 260; required the area of the other. *Ans.* $346\frac{2}{3}$.

10. The major and minor axes are 30 and 18, and the ordinate 12; what are the abscisses? *Ans.* 10 and 40.

324 : 900 :: 225 to the square of the distance of the ordinate from the centre, viz. 625, the square root of which is

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25, which being added to and subtracted from the semi-major axis, gives the abscisses.

11. The major and minor axes are 24 and 21, and the ordinate 14; what are the abscisses? *Ans.* 32 and 8.

12. The major axis is 15, and an ordinate 6, and the two abscisses 20 and 5; what is the minor axis? *Ans.* 9.

13. The major axis is 36, the ordinate 21, and the abscisses are 12 and 48; find the minor axis. *Ans.* 31.5.

14. The minor axis is 18, the ordinate 12, and the less abscis 10; what is the major axis? *Ans.* 30.

15. The minor axis is 45, the less abscis 30, and the ordinate 30; required the major axis. *Ans.* 90.

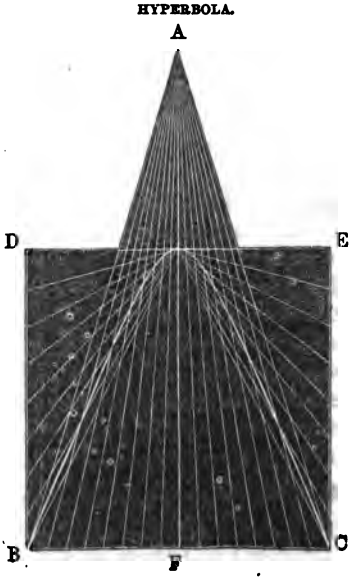
16. The major and minor axes of an hyperbola are 30 and 18, the ordinate 12, and the less abscis 10; what is the length of the arc? *Ans.* 15.66.

17. The major and minor axes of an hyperbola are 105 and 63, and a double ordinate 84, and the less abscis 35; what is the length of the whole arc? *Ans.* 109.637.

18. In an hyperbola, the major and minor axes are 15 and 9, and the less abscis 5; what is the area?

Ans. 37.919.

19. The major and minor axes are 20 and 12, and the less abscis $6\frac{2}{3}$; find the area. *Ans.* 67.408.



To construct an hyperbola:—Draw the rectangle DBCE, making BC equal to the base of the hyperbola, and BD equal to its abscisss; then, from the centre of the

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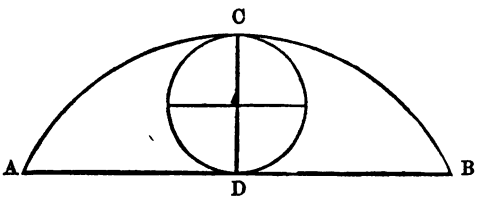
base, and perpendicular to it, draw AF, equal to the height of the cone from which the hyperbola is supposed to be cut; then divide the base, BC, into an even number of equal parts, and from each of these divisions draw lines meeting each other at A. Then divide the sides of the rectangle, DB and EC, each into half as many equal parts as there are divisions in the base, and from each of these divisions draw lines meeting at the centre of the line DE, which point is the vertex of the hyperbola. From said vertex, draw the curve through the points where the lines meeting at A cross those meeting at the vertex, and the curve will be an hyperbola.

¶ 31. CYCLOID.



The cycloid is a figure generated by the motion of a point in the circumference of a circle as it rolls along on a plane, or on a straight line.

The line AB, which is called the *base* of the cycloid, is equal to the circumference of the generating circle; and CD, which is called the *axis* of the cycloid, is equal to the diameter of the generating circle; and the whole length of the cycloidal arc, or curve, ACB, is equal to four times the diameter of the generating circle; and the whole area is equal to three times the area of the generating circle. Therefore, to find the area:—

Multiply the square of the diameter by 2.3562.

If a heavy body descends (by the force of gravity) the arc of an inverted cycloid, the velocity which it acquires is exactly

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proportional to the length of the arc; so that, from whatever point in the arc it may begin to fall, it will arrive at the lowest point in precisely the same time.—A body will descend the cycloidal arc, by the force of gravity, so as to accomplish the passage from B to C, in the inverted cycloid, in less time than by moving in a straight line, or in any other path whatever.—A pendulum made to vibrate in the arc of a cycloid, will move with uniform velocity through every point of the arc.

EXAMPLES.

1. What is the area of a cycloid, whose generating circle is 87.696 feet in circumference? and what is the length of the curve? *Answers*, 339.28 acres; and 48 feet.

2. What is the area of a cycloid generated by a circle 3 feet in diameter? and of one whose generating circle is 4 feet in diameter? *Ans.* 21.19; and 37.7.

To solve this problem by the sliding rule:—Place 2.3562 on C over 1 on D, and over the diameter of the generating circle found on D will be found the area on C. Thus, over 3 you will find 21.19, and over 4, 37.7 square feet.

3. What is the area of a cycloid, whose generating circle is 13 rods in diameter? of one whose generating circle is 20 rods in diameter? and of one whose generating circle is 24 rods in diameter? *Ans.* 2.5; 5.88; and 8.5 acres.

Place 3 on the line C over 14.28 on D, and over the diameter of the generating circle found on D, will be found the area in acres, on the line C. Thus, over 13 you will find 2.5; over 20, 5.88; and over 24, 8.5 acres.

¶ 32. IRREGULAR BOUNDARY.

To find the area of a field, or piece of land, bounded by an irregular line, as a winding creek:—

Draw a straight line as near to the irregular boundary as can

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be conveniently done; and then from all the points in the irregular boundary nearest and most remote from the right line, DC, *measure the length of the perpendiculars, or ordinates,* AD, *rs, np, mt,* &c., *falling on the right line,* DC; *and divide the sum of these perpendiculars by their number, and the quotient will be the mean or average width nearly, which being multiplied by the length of the base,* DC, *will give the area.*

If, however, any of the curves be much greater than the others, or should the whole length of the irregular boundary be a curve analogous to the hyperbolic, or parabolic, or cycloidal curve, draw a straight line, as in the above figure, and from the extremities of the curve draw ordinates to the straight line, and between these draw several equidistant ordinates; then the sum of all the ordinates, divided by their number, will give the average breadth, nearly, which, multiplied by the length of the base line, will give the area.

When the curve terminates at both extremities on the base line, divide the sum of all the equidistant ordinates by their number less one, and the quotient will be the average breadth of the curvilinear area.

EXAMPLES.

1. If the base line DC be 25 rods, and the sum of 11 ordinates be 178 rods, what is the area? *Ans.* $404\frac{6}{11}$ rods.

2. What is the area of a space bounded by a parabolic curve on one side, and the opposite side by a right line, the ordinates at the extremities of the curve being 14 and 18, and two other ordinates, equidistant from these and each other, being 15 and 16, and the base 36? *Ans.* 567.

3. A curve terminates at its extremities on a right line, and 5 equidistant ordinates are 12, 20, 26, 30, and 24, and their common distance is 14; required the area. *Ans.* 1568.

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MENSURATION OF SOLIDS.

A *solid* is a figure that has length, breadth, and thickness. The *boundaries* of solids are surfaces. A surface, no part of which is plane, is called a *curved surface.*

Any solid bounded by planes, is called a *polyhedron.* When the solid is contained by four planes, it is called a *tetrahedron;* by six, a *hexahedron;* by eight, an *octahedron;* by twelve a *dodecahedron;* and by twenty, an *icosahedron.*

The planes that bound or contain any polyhedron, are called its *sides,* or *faces;* and the lines bounding its sides, its *edges;* the edge, however, in common language, is called the side.

A prism is a solid contained by planes, of which two are opposite, equal, similar, and parallel, and the other sides are parallelograms. According as the two opposite and parallel planes are triangles, rectangles, or polygons, the prism is said to be *triangular, rectangular, square,* or *polygonal;* and the perpendicular distance between these two sides is called the *altitude* of the prism. When the lateral edges are perpendicular to the end or base, it is called a *right* prism, and in other cases an *oblique* prism; and a line joining the centres of the terminating planes, or parallel sides, is called the *axis* of the prism.

The *unit* of measure for solids is a *cube,* the length of whose edge is the *lineal unit;* and the number of cubic units in any solid, is called its *volume,* or *solid contents,* or *cubic contents.*

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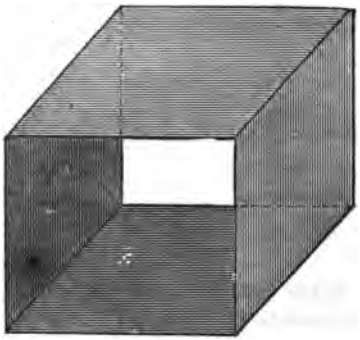
¶ 33. CUBE.



A *cube* is a solid comprehended under six equal sides, each side being an exact square.

To find the solidity of a cube:—

Cube one of its edges, or the lineal side.

To find the distance between the opposite corners of the cube:—

Extract the square root of three times the square of one of its edges.

To cube any number by the sliding rule:—

Place the number on C over 1 *on the line D; then over the number found on D, will be found its cube, or third power, on the line C.*

For the method of extracting the cube root by the sliding rule, see ¶ 11.

EXAMPLES.

1. How many solid inches in a cube whose edge is 12 inches? in one whose edge is 20 inches? in one whose edge is 30 inches? and in one whose edge is 35 inches?

Answers, 1728; 8000; 27000; and 42875.

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2. What is the superficial area of a cube 20 inches on each side, or edge? and how many bushels will a cubic box of this size contain?

Ans. 16⅔ square feet; capacity, 3 bushels, 2 pecks, and 7 quarts.

3. How many solid feet, and how many bushels in a cubic box 24 inches on a side?

Answers, 8 cubic feet; and the capacity is 6 bushels, 1 peck, and 6 quarts, nearly.

The square root of 2150.42, the number of solid inches in a bushel, is 46.37, nearly. Therefore a cubic or square box 46.37 inches on a side and one inch in depth, will hold a bushel; and if the box be two inches deep it will hold two bushels, and if three inches in depth it will hold three bushels, &c.' And the height or depth of a box (the bottom being square) being given, its capacity will be as the square of its side.

Therefore, if we place the side of a cube in inches, or the depth of any square box in inches, found on C, over 46.37 on D, then over the side found on D, will be found its capacity in bushels on the line C. Thus, in example 2d, if we place 20 over 46.37 on D, over 20 found on D will be found 3.71 bushels on the line C; and, if we place 24 on C over 46.37 on D, over 24 on D will be found 6.45 bushels on C.

The square root of $\frac{2150.42}{12}$ is 13.387, which is the side of a square, which, if one foot in depth, will contain one bushel. 13.387 is therefore the gauge point for bushels, when the depth of a square box is given in feet. Thus, in example 3d, the side of the cube being 2 feet, place 2 on C over 13.387 on D, and over 24 on D will be found 6.45 bushels on C. As 13.387 is an important gauge point, a *dot* or mark should be made on the rule under 13.387. See ¶ 22, under example 14.

4. How many bushels, and how many ale, and how many wine gallons, will a cubic vessel hold, its lineal side being 3 feet?

Answers, 21.701 bushels; 202 wine, and 165.45 ale gallons.

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To find the number of wine and ale gallons by the sliding rule:—Place the altitude or depth of any square vessel in inches over the square roots of the number of cubic inches in an ale or in a wine gallon; and then, over the side in inches found on D, will be found the number of gallons on C. Thus, place 36 on C over 15.2 on D, and over 36 found on D will be found 202, the number of wine gallons; and place 36 on C over 16.79 on D, and over 36 found on C will be found the number of ale gallons, viz. 165.45.

5. How many wine barrels of 31½ gallons will a cubic cistern hold, which is 50 inches deep?

Ans. 17.21 barrels, nearly.

6. How many ale barrels and how many coal bushels of 40 quarts each, will a cubic vessel hold, whose side is 45 inches?

Answers, 33.8 bushels, and 9.07 barrels.

The gauge point for wine barrels (the side of the cubic, or depth of the square vessel, taken in inches, being placed over the gauge point) is the square root of 7276.5, viz. 85.3, nearly; and the gauge point for ale barrels of 36 gallons is 100.75, and for coal bushels of 40 quarts, 51.84. Consequently, to solve the above examples by the sliding rule:—Place the side of the cube over the respective gauge points on the line D, and over the given side in inches will be found the required contents.

7. The side of a cubic cistern is 4½ feet; required its contents in feet, in wine and ale barrels, and in bushels of 32 and 40 quarts.

Answers, 91.13 feet; 21¾ wine, and 15.62 ale barrels; 73.45 and 58.76 bushels.

8. What is the distance between the opposite corners of a cube whose side is 15 feet? *Ans.* 26 feet, nearly.

TIMBER MEASUREMENT.

The measurement of timber will be found fully illustrated in this and the following sections. The methods here given are by far the most expeditious, whilst they are strictly accurate.

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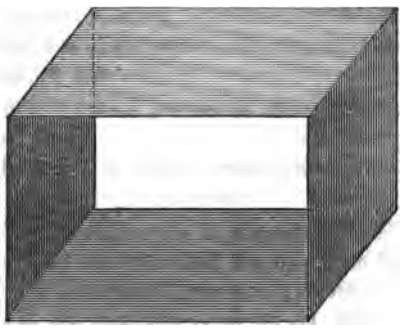
The *old* method of measuring round and rectangular timber, by means of the *girt*, has been passed over as totally unworthy of notice, in consequence of its inaccuracy, as usually practised.

To find the content of square timber:—

Multiply the square of the side by the length. Or, if the length be taken in feet, and the side of the stick in inches:—*Multiply the square of the side in inches by the length in feet, and divide the product by* 144, *and the quotient will be the content in cubic feet.*

The method of computing the content of square timber, by the sliding rule, will be found in the following paragraph; and the method of finding the content of round timber will be found fully illustrated in paragraphs 36 and 37.

¶ 34. PARALLELOPIPED.



A *parallelopiped* is a rectangular prism comprehended under six planes, the opposite planes being equal and parallel.

To find the solidity of a parallelopiped:—

Find the continued product of the length, breadth, and depth; that is—multiply the breadth by the thickness, and that product by the length.

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EXAMPLES.

1. What is the solid contents of a block of granite 25 feet long, 4 broad, and 3 thick? *Ans.* 300 solid feet.

2. Find the solidity of a stick of timber 15 feet in length, 40 inches broad, and 10 inches thick.

Ans. 41.5 cubic feet.

To solve this question, and all similar examples by the sliding rule:—*First find the side of a square whose area is equal to the area of the end of the stick or prism,* (as directed under example 11, ¶ 15;) *then place the length in feet over* 12 *on D, and over the side of the equal square found on D, will be found the cubic contents in feet on the line C.*

Thus, place the breadth 40 inches found on C, over 40 on D, and under 10, the thickness, found on C, we find 20, the side of the equal square; then, having placed the length, 15 feet, over 12 on D, over 20 found on D we find 41.5, the cubic contents.

3. Find the number of cubic feet in a rectangular stick of timber 5 feet in length, and 35 inches broad, and 25 inches thick. *Ans.* 30 feet, nearly.

4. There are 5 sticks of timber, each 8 feet in length, and the ends square, one 10 inches, one 12 inches, one 17, one 19, and one 24 inches square; what is the solid contents of each? *Ans.* 5.54; 8; 16; 20; 32 feet.

Having placed the length of the sticks, viz. 8 feet, over 12 on D, over 10 on D you will find 5.54 feet, the contents of the stick 10 inches square; and over 12 you will find 8 feet; over 17, 16 feet; over 19, 20 feet; and over 24, 32 feet. Now, since the content of any square stick, or square prism, is found on C over the side in inches on D, when the length in feet stands over 12 on D, we may reverse the case above, and find the length of a square stick which shall contain any number of feet, the side of the square end being given; thus:—*Place the number of feet which the prism is required to contain, found on C, over the side of the square in inches on D, then over* 12

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found on D, will be found the length of the prism in feet on the line C.

5. What is the length of a square stick of timber which contains 60 cubic feet, the side of the square being 40 inches?

Ans. 5.4 lineal feet.

6. How many bushels will a box hold, which is 6 feet in length, and the end 17.25 inches square? *Ans.* 10 bushels.

To solve this example by the sliding rule, see ¶ 33, under example 3d.

7. A cubic box is 25 inches square, and it holds 30 bushels; what is its length? *Ans.* 8.6 feet, nearly.

To solve this and like examples by the sliding rule:—Place the number of bushels on C, over the side of the square in inches on D, and over 13.387, the gauge point for bushels on D, will be found the length of the box in feet.

8. How many bushels will a box hold, which is 6 feet 6 inches long, 22 inches high, and 33 inches wide?

Ans. 26.4 bushels, nearly.

To solve this example by the sliding rule:—Find the side of a square box of equal capacity, as directed above; then place the length over 13.39, and over the side of the equal square box, viz. 27 inches, will be found 26.4 bushels.

9. A sleigh-box is 36 inches wide and 25 inches high, and its capacity is 40 bushels; its length is required.

Ans. 8 feet, nearly.

10. How many wine gallons will a vat hold, whose dimensions are 40, 60, and 20 inches? *Ans.* 207.78 gallons.

The gauge point for this example is 15.2 on D.

11. How many ale barrels will a vat hold, whose dimensions are 45, 60, and 40 inches? *Ans.* 10.66, nearly.

For the gauge point, see ¶ 33, under example 6.

12. How many coal bushels of 38 quarts each, will a box hold, that is 10 feet long, 40 inches wide, and 30 high?

Ans. 56.39 bushels.

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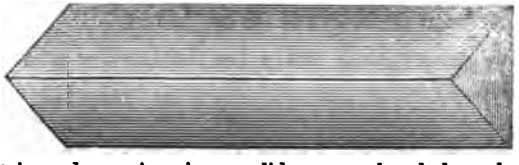
13. Find the number of cubic yards in a rectangular block of sandstone, the length of which is 16 feet, its breadth 9 feet, and height 6 feet 9 inches. *Ans.* 36 yards.

14. *The common dimensions of a brick* are 8 inches in length, 4 in width, and 2 in thickness; required the number of bricks in a cubic foot. *Ans.* 27 bricks.

15. How many bricks of the usual size will be required to build a wall 30 feet long, a brick and a half thick, and 15 feet in height, allowing that the mortar occupies one-ninth of the space occupied by the wall? *Ans.* 10,800 bricks.

¶ 85. TRIANGULAR PRISM.



A *triangular prism* is a solid comprehended under five planes, the sides being parallelograms, and the ends equal triangles.

To find the solidity of any prism:—

Multiply the area of the end, or its base, by its length, or height, and the product will be its cubic contents.

EXAMPLES.

1. What is the solidity of a triangular prism, whose length is 10 feet 6 inches, and one side of its base 14 inches, and the perpendicular on it from the opposite angle 15 inches?

Ans. 7.656 feet.

2. There is a triangular prism, the ends of which are equilateral triangles 30 inches on a side, and the length of the prism is 24 feet; required its contents in feet and in bushels.

Answers, 65 feet, and 52.2 bushels, nearly.

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To find the number of feet in the equilateral triangular prism: —*Place its length in feet and decimal parts of a foot over* 18.234 *on D, and over the side* (30 inches) *found on D, will be found its solidity in cubic feet on C.* See ¶ 19, under examples 5, 6, and 7.

To find the gauge point for bushels in an equilateral triangular prism:—Divide $\frac{1}{12}$ of 2150.42 by .4330127, and extract the square root of the quotient; and said root, viz. 20.345, will be the required gauge point, when the length of the prism is given in feet. Consequently, place the length in feet, viz. 24 on C, over 20.345 on D, and over the side of the base in inches found on D, will be found the contents in bushels on C.

3. The side of the base of an equilateral triangular prism is 120 inches; required its length, its solidity being 100 cubic feet. *Ans.* 2.3094 feet.

4. What is the solidity of a rhombic prism 18 feet in length, the side of the base being 26 inches, and the perpendicular distance between the parallel sides 15 inches?

Ans. 48.5 cubic feet.

5. What is the content of a rhomboidal prism 40 feet in length, the longest edges of the base being 38 inches, and the perpendicular between them 7.6 inches? *Ans.* 80.4 cubic feet.

See the ends of the rhombic and rhomboidal prisms in paragraphs 16 and 17. See ¶ 34, under example 2d.

6. What is the solidity of a regular pentagonal prism 7 feet in height, the edge or side of the base being 18 inches?

Ans. 27.1 cubic feet.

The gauge points for the pentagon, hexagon, and octagon, may be found on the last page of ¶ 21. In the above example, place the altitude of the prism over 9.15 on D, and over the side in inches found on D, will be found the contents in feet on C.

7. What is the solidity of a regular octagonal prism, whose altitude is 10 feet, and the edge of its base 30 inches? and how many bushels will it contain?

Answers, 301.794 cubic feet; and 242 bushels and 2 pecks, nearly.

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The gauge point for feet is 5.46, and for bushels 6.092 on D.

8. The edge of the base of an octagonal prism is 20 inches, and its contents equal to 40 bushels; required its length.

Ans. 3.72 feet, nearly.

To solve the last example by the sliding rule:—Place 40 on C over 20 on D, and over the gauge point for bushels, viz. 6.092, will be found the length or altitude of the prism.

9. What is the solidity of a regular hexagonal prism, whose altitude is 5 feet, and the edge of its base 9 inches?

Ans. 7.28 feet, nearly.

The gauge point is 7.45 inches on D.

To find the surface of any prism:—

Multiply its perimeter by the length of its lateral edge, and the product will be its lateral surface, to which add double the area of the base, and the sum will be the whole surface of the solid.

EXAMPLES.

1. What is the surface of an oblique prism 26 feet long, the perimeter of a section perpendicular to one of its lateral edges being 19 feet, and its base a rectangle 6 feet long and 4 broad?

Ans. 542 square feet.

2. What is the surface of a right rectangular parallelopiped, whose length is 36 feet, breadth 10 feet; and thickness 8 feet?

Ans. 1456 square feet.

3. The length of a rectangular cistern within is 3 feet 2 inches, the breadth 2 feet 8 inches, and height 2 feet 6 inches; required the internal surface, and the expense of lining it with lead at 4 cents per lb., the lead being 7 lbs. weight per square foot. *Ans.* $37\frac{11}{18}$ square feet, and $\$10.53\frac{1}{9}$.

4. What is the surface of a right triangular prism, its length being 20 feet, and the sides of its base 6, 8, and 10 feet?

Ans. 528 feet.

5. What is the surface of a regular pentagonal prism, whose length is $32\frac{1}{2}$ feet, and a side of the base $6\frac{1}{4}$ feet?

Ans. 1150.087 feet.

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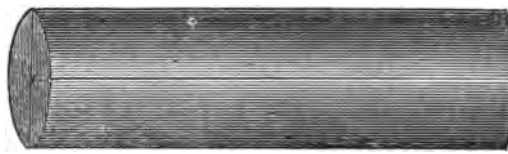
6. What is the surface of an oblique regular hexagonal prism, a side of the base being 10 inches, its length 20 feet, and its perimeter perpendicular to its lateral edges $4\frac{1}{2}$ feet?

Ans. 93.6084 feet.

¶ 36. CYLINDER.



A *cylinder* may be regarded as a prism of an infinite number of sides, the ends being circles.

To find the solidity of a cylinder:—

Multiply the area of the end, or base, by the length, or altitude. Or, multiply the length by double the square of one-fifth of the circumference, and the product will be the solidity nearly.

EXAMPLES.

1. What is the solidity of a cylinder 20 feet long and 24 inches in diameter? *Ans.* 62.832 cubic feet.

For the method of finding the gauge points for the cylinder, see examples under the circumscribing and inscribed square, ¶ 22. To solve the above example by the sliding rule:—Place the length of the cylinder in feet over 13.54 on D, and over the diameter in inches found on D, will be found the solidity in cubic feet on the line C.

2. If a square prism be hewn or cut from a cylinder 20 feet long and 24 inches in diameter, what will be the side of the square? and how many solid feet will the prism contain?

Answers, 16.97 inches; and 40 feet.

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3. What is the content of a cylinder 55 feet in length and 20 inches in diameter? *Ans.* 120 feet, nearly.

4. Find the solidity of a cylinder, the height of which is 25 inches, and its diameter 15 inches. *Ans.* 2.5566 feet.

5. The circumference of the base of an oblique cylinder is 20 feet, and its perpendicular height is 19.318; what is its volume? *Ans.* 614.93 feet.

6. How many bushels will a cylinder contain, which is 7 feet long and 30 inches in diameter? *Ans.* 27½ bush. nearly.

The gauge point for bushels, if the length or altitude of the cylinder be taken in inches, is 52.32 on D; or, if the length be taken in feet, the gauge point is 15.11. Therefore, place 7 over 15.11 on D, and over the diameter, 30 inches, found on D, will be found the number of bushels on C.

7. A bushel measure is 18.5 inches in diameter and 8 inches deep; how many solid inches does it contain?

Ans. 2150.42 inches.

8. How many wine, and how many ale gallons will a round vessel hold, whose diameter is 30 inches, and depth 23 inches?

Answers, 54 wine, and 44¼ ale gallons.

The gauge points for this example will be found in ¶ 22, examples 10 and 11.

9. How many barrels of wine and how many of ale will a cistern hold, whose diameter is 70 inches and altitude 60 inches?

Answers, 31.75 wine, and 22.8 ale, nearly.

For the gauge points for wine and ale barrels, see ¶ 22, examples 13 and 14.

10. What must be the length of a cylinder 30 inches in diameter in order that it may hold a barrel of wine? and what its length in order that it may contain a barrel of ale?

Answers, 10.29 and 14.50 inches, nearly.

11. A cylinder contains 44 cubic feet, and its diameter is 30 inches; required its length. *Ans.* 9 feet, nearly.

12. What must be the length of a cylinder 20 inches in diameter, in order that it may contain 40 ale gallons?

Ans. 35.87 inches.

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To solve the last three examples by the sliding rule:—Place the contents over the given diameter in each example, and over the respective gauge points will be found the answers required.

Thus, place 31½, the number of wine gallons in a barrel, over 30 on D, and over 17.15, the gauge point for wine gallons, will be found 10.29 inches, the answer required; and place 36, the number of ale gallons in a barrel, over 30 on D, and over 18.95 will be found 14.50 inches; and place 44 over 30, and over 13.54, the gauge point for round timber, will be found 9 feet.

13. How many solid feet, how many bushels, and how many wine hogsheads, of 63 gallons each, will a cylinder hold, whose diameter is 18 inches, and length 15 feet?

Answers, 26.6 feet; 21¼ bushels; 3.15 hogsheads.

The gauge point for a wine hogshead, the length being taken in feet and decimal parts of a foot, is 39.29 inches.

¶ 87. MILL LOGS.

TO GAUGE MILL LOGS, THE DIAMETER OF A STANDARD LOG BEING GIVEN:—

Place 1 *on C* (calling the 1 one log) *over the diameter of the standard log on D; then over the diameter of any log found on D, will be found its ratio to the standard log.* In some states, a log 19 inches in diameter is called a *standard* log; and in some states, a log 22 inches in diameter is called a *standard* log; and in some, 24 inches is the established diameter.

EXAMPLES.

1. Calling 19 inches the diameter of a *standard* log, it is required to find the ratio of the following logs to the standard log, (that is, to find what they will count in buying and selling;) one 14 inches in diameter, one 12, one 9½, one 17, one 25, one 38, and one 60 inches in diameter.

Answers in order,—.542; .4; ¼; .8; 1.72; 4; and 10 logs.

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This method gives the exact ratio of the several logs to the standard log. Thus, a log 38 inches in diameter counts 4, its solid contents being exactly 4 times greater than the contents of a log 19 inches in diameter; and a log 60 inches in diameter counts 10, or 9.97, its solid contents being nearly 10 times that of a log 19 inches in diameter. The value of logs designed for boards, does not, however, depend wholly on the number of cubic feet which they contain, for the reason that wide boards are more valuable than narrow ones in proportion to the number of feet which they contain, and likewise, because there is generally less loss of timber in sawing large logs into boards than small ones, in proportion to their cubic contents.

2. Assuming the diameter of a standard log to be 22 inches, find the ratio of the following logs to the standard: one 20 inches, one 30 inches, and one 37½ inches in diameter.

Answers, .82; 1.85; and 2.9.

3. If a standard log, 19 inches in diameter, be valued at 90 cents, what will be the value of the following logs: one 15 inches in diameter, one 20 inches, one 25 inches, and one 31 inches in diameter?

Answers in order,—$0.56; 0.997; 1.56; and 2.40.

Place the value of the standard log, found on C, over its diameter on D, and over the diameter of any log found on D, will be found its value on C.

A log 13 feet in length and 19 inches in diameter at the smaller end, will make 200 feet of inch boards; therefore, if we place 200 on C, over 19 on D, over the diameter of any other log, (taken in inches, and of the same length,) will be found nearly the quantity of inch boards which may be cut from it.

4. How many feet of inch boards will each of the following logs make, the length of each being 13 feet, and their diameters, 15 inches, 20 inches, 22 inches, and 28½ inches?

Answers, 123; 221; 267; and 450 feet.

To find the number of feet of inch boards which any log will make:—

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If the log be 2 feet in diameter, or less than two feet, allow 2 inches on four sides for the thickness of the slabs, and one-fifth for saw-calf, and 1 board for wane; but if the log is more than 2 feet in diameter, allow 3 inches for the thickness of each of the four slabs, and one-fifth for saw-calf, and 2 boards for wane. If, however, the logs are very straight and smooth, the slabs may be thinner.

EXAMPLES.

1. How many feet of inch boards will a log make whose length is 16 feet, and diameter, at the smaller end, 20 inches?

Ans. 255 feet.

After the log has been slabbed on four sides it will be 16 inches thick; then, from 16 subtract $\frac{1}{5}$ of 16 for saw-calf, and the remainder is 13 nearly, which is the number of boards the log will make; and allowing 1 board for wane, there will be 12 boards left, each 16 inches wide, and each containing $21\frac{1}{4}$ square feet: consequently, the log will make 255 feet of boards.

2. How many feet of inch boards in a log 16 inches in diameter, and 14 feet long? *Ans.* 112 feet.

3. How many feet of boards will a log make, its length being 13 feet, and diameter 31 inches? *Ans.* 487 feet.

See the rule for gauging boards, ¶ 15, example 4. Market boards are usually a little less than one inch in thickness; and consequently, the number of feet of market boards in a log will be greater than the number of feet of inch boards. To find the number of feet of market boards, in any log, allow one-eighth for saw-calf, and apply the above rule for inch boards with this difference.

4. How many feet of market boards will a log make, its length being 20 feet, and its diameter 46 inches?

Ans. 2,200 feet.

The following table exhibits the number of feet of market boards in any log, whose diameter at the smaller end ranges from 15 to 36 inches, and its length from 10 to 15 feet.

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LOG TABLE.

Diameter in inch.	10 ft. in length.	Diameter in inch.	11 ft. in length.	Diameter in inch.	12 ft. in length.	Diameter in inch.	13 ft. in length.	Diameter in inch.	14 ft. in length.	Diameter in inch.	15 ft. in length.
15	90	15	99	15	108	15	117	15	126	15	135
16	100	16	110	16	120	16	130	16	140	16	150
17	125	17	137	17	150	17	160	17	175	17	187
18	155	18	170	18	186	18	201	18	216	18	232
19	165	19	176	19	198	19	214	19	230	19	247
20	172	20	189	20	206	20	233	20	246	20	258
21	184	21	202	21	220	21	263	21	256	21	276
22	194	22	212	22	232	22	283	22	294	22	291
23	219	23	240	23	278	23	315	23	332	23	333
24	250	24	276	24	300	24	325	24	350	24	375
25	280	25	308	25	336	25	364	25	392	25	420
26	299	26	323	26	346	26	375	26	404	26	448
27	327	27	367	27	392	27	425	27	457	27	490
28	360	28	396	28	432	28	462	28	504	28	540
29	376	29	414	29	451	29	488	29	526	29	564
30	412	30	452	30	494	30	535	30	576	30	618
31	428	31	471	31	513	31	558	31	602	31	642
32	451	32	496	32	541	32	587	32	631	32	676
33	490	33	539	33	588	33	637	33	686	33	735
34	532	34	585	34	638	34	691	34	744	34	798
35	582	35	640	35	698	35	752	35	805	35	863
36	593	36	657	36	717	36	821	36	836	36	889

In some states a *standard* board is 12 feet long, 12 inches wide, and 1 inch thick; and this is what is generally to be understood by a *standard* board.

¶ 38. PYRAMIDS.

A pyramid is a solid having any rectilineal figure for its base, and its other sides are triangles, whose vertexes all meet in one point, called the *vertex* of the pyramid. The *altitude* of a pyramid is a perpendicular from its vertex on the plane of the

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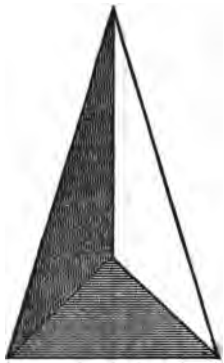
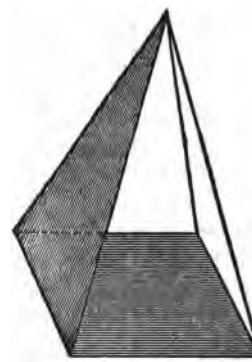
TRIANGULAR PYRAMID.



QUADRANGULAR PYRAMID.



base. The pyramid is said to be *triangular, quadrilateral, polygonal*, &c., according as its base is a triangle, a quadrilateral, a polygon, &c. When the base of the pyramid is regular, a right line joining its centre and the vertex is called the *axis* of the pyramid; and when the axis is perpendicular to the base it is called a *right* pyramid; but if it be not perpendicular to the base, the pyramid is said to be *oblique*.

The pyramid is just one-third of its least circumscribing prism. Therefore,

To find the solidity of a pyramid:—

Multiply the area of its base by one-third of its perpendicular altitude, and the product will be the solidity.

To find the surface of a pyramid:—

If it be a *right* pyramid, ***multiply the perimeter of the base by the altitude, or apothegm, of one of its slanting sides, and add half the product to the area of the base;*** but if the pyramid be ***oblique, find the areas of the lateral triangles separately, and to their sum add the area of the base.***

To find the altitude of a pyramid, its base and solidity being given:—***Divide three times its solidity by the area of its base, and the quotient will be the altitude.*** Or, ***three times the solidity divided by the altitude of a pyramid, will give the area of its base.***

If we take two square pyramids, whose bases are equal to

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each other, and the sides of the base equal to the lateral edges, and place the base of one of the pyramids upon that of the other, a solid will thus be formed, which is comprehended under eight equilateral triangular faces. This solid is called the OCTAHEDRON.

To find the solidity of an *octahedron* :—Multiply the square of one of its edges by one-third of the distance between two opposite vertexes. Or, multiply the cube of one of its edges by 0.4714045 for the *solidity;* and multiply the square of one of its edges by 3.4641016 for the *surface* of its eight faces. To find the distance between the vertexes of an octahedron :—Extract the square root of twice the square of one of the edges.

When the base of a triangular pyramid is equilateral, and the lateral faces are likewise equilateral and equal to the base, the pyramid is called a TETRAHEDRON.

To find the solidity of a *tetrahedron* :—Multiply the cube of one of its edges by 0.1178513; and multiply the square of one of its edges by 1.7320508 for the surface of its four faces.

The distance of the centre of the base of the tetrahedron from one of the angles of the base may be found by extracting the square root of one-third of the square of one of the sides of the base; and the square root of two-thirds of the square of one of the sides will be the altitude of the tetrahedron.

EXAMPLES.

1. What is the solidity of a square pyramid, each side of its base being 3 feet, and its altitude 10 feet? *Ans.* 30 feet.

2. What is the solidity of an equilateral triangular pyramid, each side of the base being 3 feet, and its altitude 30 feet?

Ans. 38.97114 feet.

3. Find the solidity of a square pyramid, each side of the base being 30 feet, and its *apothegm* (that is, the perpendicular height of one of its lateral triangles) 25 feet.

Ans. 6,000 feet.

4. What is the solidity of a pentagonal pyramid, each side of the base being 4 feet, and its altitude 30 feet?

Ans. 275.276 feet.

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5. How many solid feet, and how many bushels in a pyramid, its altitude being 9 feet, and its base being 4 feet square?

Answers, 48 feet, and 38.6 bushels.

The *gauge points* for the pyramids are the same as for the corresponding prisms. Therefore, *place one-third of the altitude of any pyramid over the gauge point for the prism, whose base is similar to that of the pyramid, and over the side found on D will be found the contents of the pyramid on C.* See ¶ 33, under example 3d.

6. What is the content of an hexagonal pyramid, the altitude being 6.4 feet, and each side of the base 6 inches?

Ans. 1.3856072 feet.

7. The hopper of a cider-mill is a square pyramid, its base being 4 feet on a side; required its length in order that it may hold 50 bushels. *Ans.* 11.66 feet.

Place three times the contents, viz. 150 bushels, over one side of the base, reduced to inches, viz. 48, and over the gauge point for bushels in a square prism, viz. 13.387, will be found the altitude of the hopper.

8. Find the surface of a square pyramid, its apothegm being 40 feet, and each side of the base 6 feet.

Ans. 516 square feet.

9. Find the surface of a pyramid whose apothegm is 10 feet, and its base an equilateral triangle, whose side is 18 inches.

Ans. 23.474278 feet.

10. The apothegm of a regular hexagonal pyramid is 8 feet, and the side of its base $2\frac{1}{2}$ feet; what is its surface?

Ans. 76.23795 feet.

11. What is the solidity of a tetrahedron, whose edge is 8?

Ans. 60.33.

12. What are the solidity and surface of a tetrahedron, whose edge is 6? *Answers,* 25.45, and 62.35.

13. Find the solidity of an octahedron, whose edge is 16.

Ans. 1930.87.

14. What are the solidity and surface of an octahedron, whose edge is 3? *Answers,* 12.728 and 31.177.

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15. What is the altitude of a tetrahedron, whose edge is 6?

Ans. 4.9, nearly.

16. What is the diameter of a globe from which you may cut an octahedron whose side is 6 inches?

Ans. 8.48528 inches.

¶ 39. REGULAR SOLIDS.

There are five regular solids, or, as they are sometimes called, Platonic Bodies, and it can be proved that no more can exist. These solids are the *tetrahedron*, *hexahedron*, or cube, the *octahedron*, *dodecahedron*, and the *icosahedron*. The first three have already been described.

The *dodecahedron* is a solid comprehended under twelve regular pentagons; and it may be considered as a solid composed of twelve pentagonal pyramids, whose bases form the surface of the solid, and whose vertexes all meet in its centre.

The *icosahedron* is a solid comprehended under twenty equilateral triangular faces; and the solid may be conceived to consist of twenty equal triangular pyramids, whose bases form the surface of the solid, and whose vertexes all meet in the centre.

Each side of a regular solid, except the tetrahedron, has an opposite face parallel to it, and the edges of these faces are also respectively parallel.

To find the solidity of a dodecahedron:—

Multiply its surface by one-sixth of the distance between two parallel sides. Or, Multiply the cube of one of its edges by 7.6631189.

To find the surface of a dodecahedron:—

Multiply the square of one of its edges by 20.6457088.

To find the perpendicular distance between two parallel planes of the dodecahedron:—Multiply one of the edges by 2.2270274.

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To gauge a dodecahedron:—Place twice the distance between two parallel planes over the gauge points for the pentagon, or for the pentagonal prism, and over the edge or side found on D, will be found the cubic contents on the line C. The gauge points are (for feet, when the dimensions are taken in feet) 0.7623; and 9.15, (for feet, when the distance between the parallel planes is taken in feet, and the side in inches;) and 10.205 for bushels, (when the distance between the parallel planes is taken in feet, and the side in inches.)

To find the solidity of an *icosahedron*:—

Multiply its surface by one-sixth of the distance between the parallel faces. Or, Multiply the cube of one of its edges by 2.181695, and the product will be the solidity.

To find the surface of the *icosahedron*:—

Multiply the square of one of its edges by 8.6602540.

To find the perpendicular distance between two parallel faces of the *icosahedron*:—

Multiply the side, or one of the edges, by 1.5115226.

To gauge an *icosahedron*:—Place ten-thirds of the distance between the parallel planes, over the gauge points for the equilateral triangular pyramid, and over the side found on D, will be found the contents on C. Or:—Place the distance *in feet* between two parallel surfaces over 11.143 on D, and over the side in inches will be found the contents in bushels; or, over 9.99, and over the side in inches will be found the contents in cubic feet; or, if we place the distance between the parallel surfaces *in inches*, over 13.98 on D, over the side in inches found on D will be found the contents in ale gallons; or, over 12.65, and over the side in inches found on D, will be found the contents in wine gallons.

EXAMPLES.

1. What is the solidity of a dodecahedron the side of which is 6? *Ans.* 1655.23.

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2. What are the solidity and surface of a dodecahedron whose side is 4? *Answers*, 490.4396, and 330.332.

3. How many feet and how many bushels are contained in a dodecahedron, the distance between the parallel faces being 5 feet, and the side 26.94 inches?

Answers, 70 bushels, and 87 feet, nearly.

4. Find the solidity of an icosahedron whose edge is 6.

Ans. 471.245.

5. What are the solidity and surface of an icosahedron, whose edge is 5? *Ans.* 272.71187, and 216.50635.

6. How many solid feet, ale and wine gallons, and bushels, in an icosahedron, the distance between its parallel faces being 24 inches, and its side 16.54 inches?

Ans. 41 wine, 33.62 ale gallons; 4.405 bushels; and 5.485 feet.

7. What is the solidity of a hexahedron, of a tetrahedron, of an octahedron, of a dodecahedron, and of an icosahedron, the edge of each being 6 inches?

Answers, 216; 25.4559; 101.823372; 1655.2336; and 471.24612 cubic inches.

8. How many times greater is a dodecahedron 12 inches on a side, than one 6 inches on a side? *Ans.* 8.

The following table exhibits the surfaces and solidities of the regular solids whose edges are 1:—

No. of sides.	Names.	Surfaces.	Solidities.
6	Hexahedron.	6.	1.
4	Tetrahedron.	1.7320508	0.1178519
8	Octahedron.	3.4641016	0.4714045
12	Dodecahedron.	20.6457088	7.6631189
20	Icosahedron.	8.6602540	2.1816950

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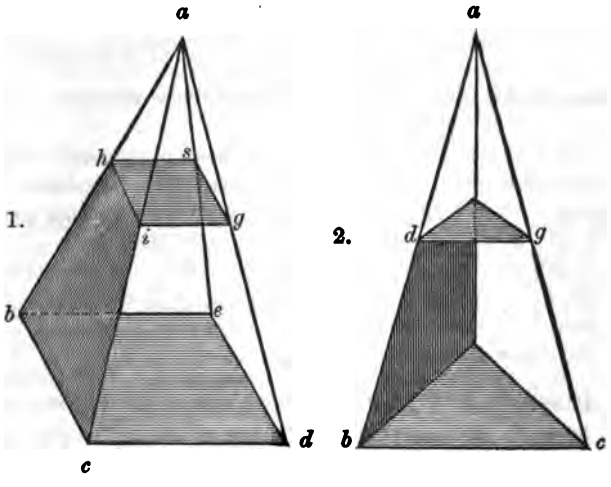
¶ 40. FRUSTUMS OF PYRAMIDS.



To find the solidity of the frustum of a regular pyramid:—

Complete the solid; and then find the solidity of the whole pyramid, *abcde*, and also the solidity of the pyramid, *ahigs*, standing on the less base; then subtract the contents of the pyramid, *ahigs*, from the solidity of the whole pyramid, *abcde*, and the remainder will be the solidity of the frustum.

To find the altitude of the pyramid when completed, say:— As the difference of the edges of the less and greater base is to the greater base, so is the perpendicular altitude of the frustum to the required altitude of the pyramid when completed.

Or,

To find the solidity of the frustum of a square pyramid:—

Multiply the edge of the greater base by the edge of the less, and to the product add one-third of the square of the difference of the edges of the less and greater base, and the sum will be the mean area, which, being multiplied by the altitude of the frus-

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tum, will give its required solidity.—Apply this rule to the frustum of the equilateral triangular pyramid, and then multiply the square mean area by 0.4330127, and the product will be the required mean area of the triangular frustum, which, multiplied by its altitude, will give the required solidity. The same rule will apply to the frustums of the regular pentagonal, hexagonal, heptagonal pyramids, &c., if we multiply the square mean areas by the *tabular areas* of the regular polygons, whose edge is 1. See ¶ 21, page 63.

The solidities of similar cones, or of similar pyramids, are to each other as the cubes of their bases, or as the cubes of their altitudes. Thus, in figure 2, the solidity of the triangular pyramid, *abc*, is to the cube of its base, *bc*, as the solidity of the pyramid, *adg*, is to the cube of its base, *dg*. Or, the solidity of the former pyramid is to that of the latter, as the cube of the base of the former to that of the latter; and the same is true of the cubes of their altitudes.

To find the solidity of the frustum of any pyramid:—

Multiply the area of the greater base by one of its edges, and the area of the less base by its corresponding edge; divide the difference of the products by the difference of these edges, and the quotient divided by 3 will give the mean area, which, multiplied by the altitude of the frustum, will give the solidity.

EXAMPLES.

1. Find the solidity of a frustum of a pyramid, the edges of the ends, or bases, being 3 feet and $2\frac{1}{2}$ feet square, and its height 5 feet. *Ans.* $37\frac{1}{12}$ feet.

2. Find the solidity of a frustum of a square pyramid, the edges of its ends being 10 and 16 inches, and its length 18 feet. *Ans.* $21\frac{1}{3}$ feet.

3. Find the solidity of a frustum of a regular hexagonal pyramid, the edges of its ends being 4 and 6 feet, and its length 24 feet. *Ans.* 1579.6302 feet.

4. Find the solidity of a frustum of a regular octagonal

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pyramid, the edges of its bases being 3 and 5 feet, and its height 10 feet. *Ans.* 788.648 feet.

5. What is the solidity of the frustum of an equilateral triangular pyramid, the sides of the greater base being 15, and of the less 6, and the height 40? *Ans.* 2026.44.

6. The altitude of a square pyramid is 66⅔ feet, and its base 15 feet on a side; it is required to find the distance from the vertex of the pyramid of a plane, which will cut off one-fifth of the solid contents. *Ans.* 38.9863 feet.

7. The altitude of an equilateral triangular pyramid is 9, and the side of its base 6; at what distance above the base must it be cut off, in order to divide it into two equal parts? *Ans.* 1.80165.

To find the surface of a frustum of a pyramid:—

When the pyramid is regular, multiply the sum of the perimeters of the two ends by the lateral length, and to half the product add the areas of the two ends, and the sum will be the surface.

When the pyramid is irregular, the lateral planes are trapezoids, and their areas may be found separately by the rule for the trapezoid, ¶ 18, and to their sum add the areas of the two ends for the whole surface.

8. Find the surface of the frustum of a square pyramid, the sides of its ends being 14 and 24 inches, and the lateral length 2 feet 3 inches. *Ans.* 19.61 feet.

9. What is the surface of the frustum of a regular pentagonal pyramid, its lateral length being 5 feet 10 inches, and the sides of its ends 10 and 15 inches? *Ans.* 34.2649 feet.

10. Find the solidity of the frustum of a regular pyramid of ten sides, the edges of its ends being 4 and 6 feet, and its length 30 feet. *Ans.* 5847.597 cubic feet.

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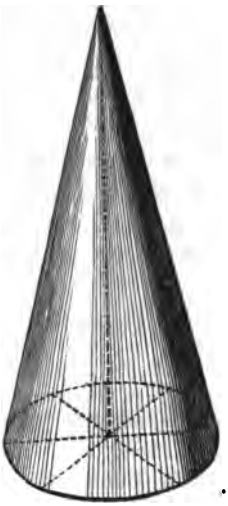
¶ 41. CONE.



A *cone* may be regarded as a pyramid of an infinite number of sides, its base being a circle, and its convex surface terminating in a point called the *vertex.*

The line joining the vertex and centre of the base is called the *axis;* and when the axis is perpendicular to the base, it is called a *right* cone; otherwise it is said to be an *oblique* cone.

A right cone is generated by the revolution of a right-angled triangle about its base, or its perpendicular.—A cone is one-third of its least circumscribing cylinder. Therefore,

To find the solidity of a cone:—

Multiply the area of its base by one-third of its perpendicular altitude.

The convex surface of a cone is the sector of a circle, whose arc is the circumference of the base, and whose radii are equal to the *slant,* or slanting height of the cone. Therefore,

To find the surface of a cone:—

Multiply the circumference of the base by one-half the slant height, and to the product add the area of the base.

In any cone or pyramid, a section or plane parallel to the base is similar to the base, and the areas of the two planes are to each other as the squares of their respective distances from the vertex.

To find the solidity of a cone by the sliding rule:—Place one-third of its altitude over the gauge points for the cylinder,

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and over the diameter of the base found on D, will be found its solidity on C. See ¶ 36, page 119.

EXAMPLES.

1. The altitude of a right cone is 30 feet, and the diameter of its base 6 feet; what is its solidity? *Ans.* 282.74 feet.

2. The altitude of a right cone is 12, and the diameter of its base 10; what is its solidity? *Ans.* 314.16.

3. Find the solidity of a cone, whose altitude is 20 feet, and the diameter of its base 30 inches. *Ans.* 32.76.

4. The altitude of a cone is 27 feet, and the diameter of its base 7 feet; find its contents in feet and bushels.

Answers, 346½ feet, and 278 bushels, nearly.

5. The altitude of a cone is 9 feet, and the diameter of its base 15 inches; find its contents in ale and wine gallons.

Answers, 22½ ale, and 27½ wine gallons.

6. The altitude of a cone is 25 feet, and the circumference of its base 20 feet; what is its solidity? *Ans.* 265.25 feet.

7. What is the surface of a cone, whose altitude is 18 feet, and the diameter of its base 5 feet? *Ans.* 162.364 feet.

8. The slant height of a cone is 40 feet, and the diameter of its base 9 feet; what is its surface? *Ans.* 629.11 feet.

9. The altitude of a cone is 120 feet, and the circumference of its base 30 inches; what is the length of an inch ribbon, which will wind round the cone from its base to its vertex, and leave a space of 5 inches between the several flexures?

Ans. 100 yards, nearly.

¶ 42. FRUSTUM OF A CONE.

To find the solidity of a frustum of a cone:—

1. Complete the solid, and proceed as directed in ¶ 40.

2. Or, *Multiply the diameter of the greater base by the diam-*

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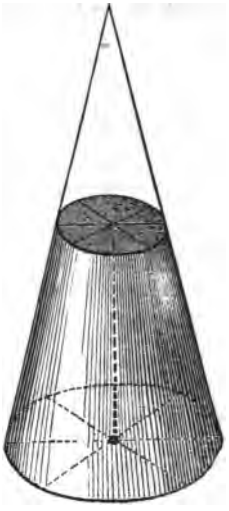
eter of the less, and to the product add one-third of the square of the difference of the two diameters, and multiply the sum by 0.7854, *and the product will be the mean area, which, multiplied by the altitude of the frustum, will give the solidity.*

CONIC FRUSTUM.



3. Or, *Multiply the sum of the squares and the product of the two diameters by* .2618, *and the product will be the mean area.*

4. Or, *To the squares of the circumferences of the two ends add the product of the circumferences, and multiply the sum by the height of the frustum, and this product by* .02652, *and the result will be the solidity.*

To find the area of the convex surface of a conic frustum:—

Multiply half the sum of the circumferences of the two ends by the slant height.

The diameters of the two ends of a conic frustum and its solidity being given, to find its altitude:—

Divide the solidity by its mean area, and the quotient will be the altitude.

EXAMPLES.

1. What is the solidity of a frustum of a cone, whose altitude is 10 feet, and the diameters of its ends 2 and 4 feet?

Ans. 73.3 feet.

2. How many cubic feet are contained in a ship's mast, whose length is 72 feet, and the diameters of its ends 1 foot and $1\frac{1}{2}$?

Ans. 89.5356.

3. How many cubic feet are contained in a cask, which is

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composed of two equal and similar conic frustums, united at their greater ends, its bung diameter being 14 inches, its head diameter 10 inches, and its length 20 inches?

Ans. 1.34838.

4. A ship's mast is 60 feet long, and the diameters of its ends 12 and 30 inches; find its superficial and solid contents.

Answers, 335.562 square, and 153.15 solid feet.

5. The bung diameter of a cask composed of two conic frustums, is 26 inches, the head diameters 20 inches each, and its length 30 inches; required its contents in ale and wine gallons. *Answers,* 44.33, and 54.33.

6. How many wine gallons will a tub hold, whose diameters are 30 and 40, and its depth 60 inches?

Ans. 251.6 gallons.

To gauge a conic frustum:—

Find the mean diameter, and then place the length of the frustum over the gauge points for the cylinder, and over the mean diameter, found on D, will be found its contents on C. The mean diameter may be found sufficiently near for most practical purposes, by adding .52 of the difference of the two diameters to the less diameter.

7. What is the convex surface of a conic frustum whose slant height is 10 feet, and the circumferences of its two ends, 5 and 15 feet? *Ans.* 100 feet.

¶ 43. THE WEDGE.

A wedge is a solid, having a rectangular base, and two opposite sides terminating in an edge. When the edge and base are of equal length, it is called a *direct* wedge; and when the base is longer than the edge, it is called an *indirect* wedge.

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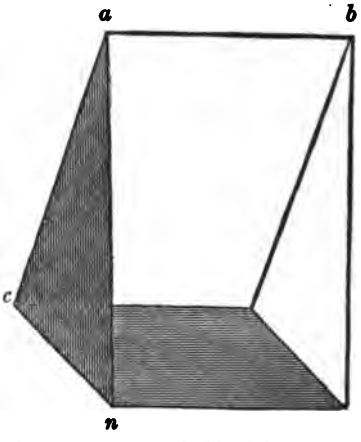
A DIRECT WEDGE.

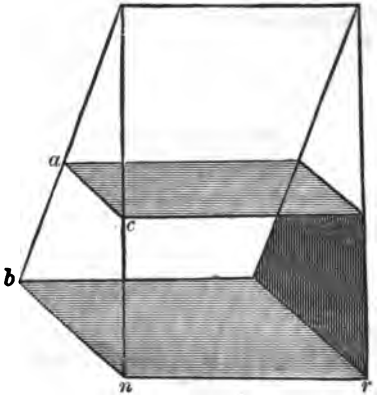


A direct wedge is equal to half of its corresponding prism of the same base and altitude. Therefore,

To find the solidity of a direct wedge:—

Multiply the area of its base by half its perpendicular altitude. Or, *Multiply the area of the triangle acn, by ab, and the product will be the solidity.*

FRUSTUM OF A DIRECT WEDGE.



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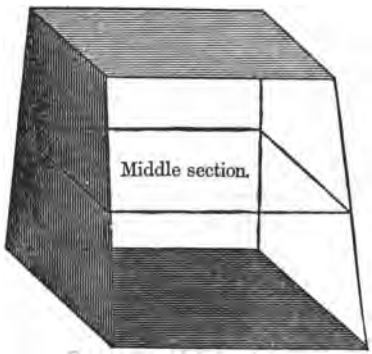
To find the solidity of a frustum of a direct wedge:—

Multiply the area of the lateral surface *abnc*, by the breadth of the wedge *nr*. Or, if the wedge be not of equal thickness throughout, multiply half the sum of the lateral surfaces by the breadth of the wedge, and the product will be the solidity, nearly.

To find the solidity of an *indirect wedge*:—

To the length of the edge add twice the length of the back, or base, and reserve this sum; then multiply the altitude of the wedge by the thickness of the back, and this product by the reserved sum, and one-sixth of this product will be the solidity.

¶ 44. PRISMOID.



To find the solidity of a prismoid:—

Add together the areas of the two ends, and four times the area of the middle section parallel to the two ends, and one-sixth of the sum will be the mean area, which, multiplied by the altitude of the prismoid, gives the solidity.

The prismoid may be regarded as the frustum of an indirect wedge. The length of the middle section of the prismoid is

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equal to half the sum of the lengths of the two ends, and its breadth is equal to half the sum of the breadths of the two ends.

EXAMPLES.

1. What is the solidity of a direct wedge, whose back is 27 inches long and 8 inches thick, and its perpendicular height 40 inches? *Ans.* 4320 inches.

2. What is the solidity of the frustum of a direct wedge, the length of the base being 40 and its breadth 10 inches, the breadth of the other end 4 inches, and its altitude 30 inches? *Ans.* 8400 inches.

3. How many bushels will a box contain, its form being that of a frustum of a direct wedge, whose base is 3 feet in length, the breadths of the two ends 16 and 10 inches, and the height 18 inches? *Ans.* 3.9174 bushels.

4. What is the solidity of an indirect wedge, whose altitude is 14 inches, its edge 21, the length of the back 32, and its thickness $4\frac{1}{2}$ inches? *Ans.* $892\frac{1}{2}$ cubic inches.

5. Find the contents of a wedge, whose base is 16 inches long and $2\frac{1}{4}$ broad, its edge $10\frac{1}{2}$ inches, and its altitude 7 inches. *Ans.* 111.56 inches.

6. The length and breadth of the base of a wedge are 70 and 30 inches, the length of the edge 9 feet 2 inches, and the altitude 34.29016 inches; what is its solidity? *Ans.* 24.8048 feet.

7. Find the solidity of a prismoid, the length and breadth of its base being 10 and 8, those of the top 6 and 5, and the height 40 feet. *Ans.* 2120 cubic feet.

8. What is the solidity of a stick of timber, the length and breadth of one end being 2 feet 4 inches, and 2 feet, and those of the other end 1 foot and 8 inches, and its altitude or length 61 feet? *Ans.* 144.592 feet.

9. Find the capacity of a trough of the form of a prismoid,

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its bottom being 48 inches long and 40 inches broad, its top 5 feet long and 4 feet broad, and its depth 3 feet.

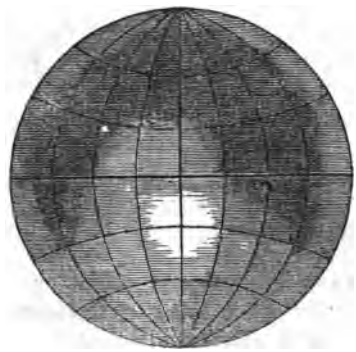
Ans. 49⅔ feet.

10. How many coal bushels, of 38 quarts each, will a box hold, its bottom being a rectangle 4 feet by 8, and the top a rectangle 6 feet by 10, and its depth 5 feet?

Ans. 153½ bushels.

To gauge a prismoid:—Find the side of a square whose area is equal to the mean area of the prismoid, and then proceed in the same manner as with the square prism.

¶ 45. A SPHERE.



A *sphere* or *globe* is a solid, comprehended under a convex surface, every point in which is equally distant from a certain point within, called the *centre*.

Any line drawn from the centre to the surface is a *radius*; and any line drawn through the centre, and terminating at the surface at both extremities, is a *diameter*.

The sphere may be generated by the revolution of a semicircle about its axis; and it may be considered as composed of

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an infinite number of cones or pyramids, whose bases form the convex surface, and whose vertexes all meet in the centre. The sphere is equal to a cone whose base is equal to the convex surface of the sphere, and its height equal to the radius of the sphere; it is likewise equal to two-thirds of its least circumscribing cylinder. Therefore,

To find the solidity of a sphere :—

Multiply the area of its convex surface by one-third of the radius of the sphere, or by one-sixth of the diameter. Or,

Find the solidity of a cylinder, whose diameter and length are equal to the diameter of the sphere, and two-thirds of it will be the solidity of the sphere. Or,

Multiply the cube of the diameter of the sphere by .5236 ; or, *when great accuracy is required, by* .5235938.

To find the greatest cube that can be cut from a sphere, its diameter being given :—

Extract the square root of one-third of the square of the diameter for the side of the cube. Or, Multiply the diameter by .57735.

To find the diameter of a sphere, its solidity being given :—

Divide the solidity by .5236, and the quotient will be the solidity of the least circumscribing cube, the cube root of which will be the diameter.

To gauge a sphere :—*Place two-thirds of the diameter over the gauge points for the cylinder, and over the diameter found on D will be found its solidity on the line C.*

The surface of a sphere is equal to the convex surface of its least circumscribing cylinder : it is also equal to four times the area of a great circle of the sphere. Therefore,

To find the surface of a sphere :—

Multiply its circumference by its diameter. Or,

Multiply the square of its diameter by 3.1416.

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EXAMPLES.

1. How many cubic inches are contained in a sphere 25 inches in diameter? *Ans.* 8181.25.

2. Find the solidity of a sphere, the diameter of which is 8½ inches. *Ans.* 321.55585 cubic inches.

3. How many cubic feet of gas will a balloon of a spherical form contain, its diameter being 50 feet?

Ans. 65449.85 feet.

4. What is the solidity, and what the surface of a sphere, whose diameter is 1? *Answers,* 0.5235988, and 3.1415926.

5. What is the diameter of a sphere whose capacity is 4 wine gallons? *Ans.* 12.08 inches.

6. How many solid feet, how many bushels, and how many ale gallons in a sphere whose diameter is 6 feet?

Answers, 113.1 feet; 91 bushels; and 695 gallons, nearly.

The gauge points for the last example may be found under the cylinder, ¶ 36; or the gauge points for the sphere, placing the whole diameter over the gauge point, may be found thus:—Divide the given number of inches (as 144 for feet, or 231 for wine, and 282 for ale gallons, &c.) by .5236, and extract the square root of the quotient.

7. What is the gauge point for solid feet in a sphere, if we place the diameter in feet over the gauge point?

Ans. 16.58 inches.

8. What is the gauge point for bushels in a sphere?

Ans. 18.5 inches.

9. What is the gauge point for wine gallons in a sphere, the diameter in inches being placed over the gauge point?

Ans. 21 inches, nearly.

10. How many wine gallons will a sphere contain, whose diameter is 12.1 inches? *Ans.* 4 gallons, nearly.

11. How many spheres 4 feet in diameter will it take to equal the contents of a sphere 8 feet in diameter?

Ans. 8 spheres.

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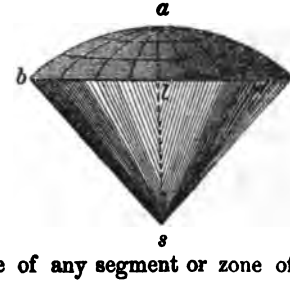
12. What is the side of the greatest cube that can be cut from a sphere 12 inches in diameter? *Ans.* 6.9282 inches.

13. How many square inches of gold-leaf will gild a sphere one foot in diameter? *Ans.* 452.39 inches.

14. What is the surface of a sphere whose diameter is 2 feet 8 inches? *Ans.* 22.34 feet.

¶ 46. SPHERICAL SECTOR.



A spherical sector bears the same relation to the sphere, that the circular sector does to the circle.

The areas of spherical surfaces are as their altitudes. Therefore,

To find the spherical surface of any segment or zone of a sphere:—

Multiply the circumference of the sphere (of which it is a part) by the height of the segment or zone, and the product will be the convex surface. Or,

Multiply the diameter of the sphere by 3.1416, and the product by the height of the segment or zone, and the product will be the area required.

To find the solidity of the spherical sector:—

Multiply the area of the convex surface by one-third of the radius of the sphere, and the product will be the solidity.

To find the solidity of a *spherical segment*:—

1. *Find the solidity of the whole sector, bacs, from which subtract the solidity of the cone bcs, and the remainder will be the solidity of the segment bac.*

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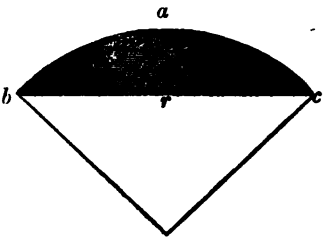
SPHERICAL SEGMENT.



2. Or, to find the solidity of a spherical segment:—

To three times the square of the radius of the base, br, add the square of the height ar, and multiply the sum by the height, and that product by .5236, and the result will be the solidity.

3. Or, *From three times the diameter of the sphere, subtract twice the height of the segment; multiply the remainder by the square of the height, and that product by .5236, and the result will be the solidity.*

EXAMPLES.

1. What is the area of the convex surface of a spherical sector, the diameter of the sphere being 10 feet, and the height of the segment *ar* 2 feet? *Ans.* 62.832 feet.

2. Find the convex surface of a spherical segment, whose height is 3 feet 6 inches, and the diameter of the sphere 10 feet. *Ans.* 109.96 feet.

3. Find the convex surface of a spherical zone, whose height is 4 inches, and the diameter of the sphere 1 foot. *Ans.* 150.7968 inches.

4. Find the convex surface of a spherical zone, the height of which is 5 inches, and the diameter of the sphere 25 inches. *Ans.* 392.7 inches.

5. Find the convex surface of a spherical segment, whose height is 9 inches, and the diameter of the sphere 3 feet 6 inches. *Ans.* 1187.525 inches.

6. What is the solidity of a spherical sector, the height of the convex part being 2 inches, and the diameter of the sphere 9 inches? *Ans.* 84.8232 inches.

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7. What is the solidity of a spherical sector, whose height is 4 feet, and the diameter of the sphere 70 feet?

Ans. 1026.256 feet.

8. Find the solidity of a spherical segment, whose height is 4 inches, and the radius of the base 8 inches.

Ans. 435.6352 inches.

9. What is the solidity of a spherical segment, the radius of whose base is 25 inches, and its height 6.75 inches?

Ans. 6787.844 inches.

10. Find the solidity of a spherical segment, the height of which is 2 feet, and the diameter of the sphere 10 feet.

Ans. 54.4544 feet.

11. Find the contents of a spherical segment in ale and wine gallons, the diameter of its base being 83.32 inches, and its height 28 inches.

Ans. 311.51 ale, and 380.29 wine gallons.

When the diameter of the segment's base and its height are given, to find the diameter of the sphere:—Divide the square of the radius of the base by the height, and to the quotient add the height. See ¶ 24, page 82.

The solidity of a spherical segment is equal to that of half of its least circumscribing cylinder, plus the solidity of a sphere whose diameter is equal to the height of the segment. Therefore,

To gauge a spherical segment:—

Find the contents of one-half of its least circumscribing cylinder, and the contents of a sphere whose diameter is equal to the height of the segment, and the sum of their contents will be the solidity of the spherical segment.

12. What is the solidity of a spherical segment, whose height is 4 feet, and the diameter of its base 16 feet?

Ans. 435.635 feet.

Place 2 feet over 13.54 on D, the gauge point for feet in a cylinder, and over 192, the diameter of the segment's base in inches, you will find 302.3 feet, the solidity of one-half the

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least circumscribing cylinder; then place 4 feet over 16.58, the gauge point for feet in a sphere, and over 48, the height of the segment in inches, you will find 33.4 feet, which, being added to 302.3, equals 435.7 feet for the solidity of the segment.

¶ 47. SPHERICAL ZONE.

A spherical zone is the frustum of a sphere formed by cutting off two segments by planes parallel to each other.

1. To find the solidity of a spherical zone:—*Subtract the solidity of the two segments from the solidity of the whole sphere, and the remainder will be the solidity of the zone.* Or,

2. *Add together the squares of the radii of the two ends, and one-third of the square of the height, which sum multiply by the height, and this product by* 1.5708, *and the result will be the solidity.*

EXAMPLES.

1. Find the solidity of a spherical zone, the diameter of the ends being 4 and 3 inches, and its height 2 inches.

Ans. 23.824 cubic inches.

2. Find the solidity of the middle zone of a sphere, the diameters of the ends being 4 feet each, and its height 6 feet.

Ans. 188.496 inches.

3. The diameters of the ends of a spherical zone are 8 and 12 inches, and its height 10 inches; what is its solidity?

Ans. 1340.416 inches.

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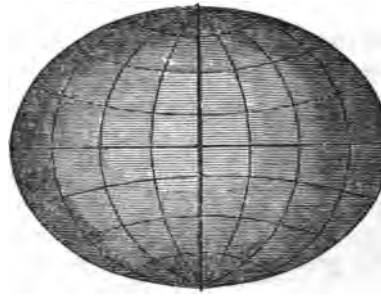
¶ 48. SOLIDS GENERATED BY THE REVOLUTION OF THE CONIC SECTIONS.

The solids generated by the revolution of the conic sections are the *ellipsoids*, (called the *oblate* and *prolate spheroids*,) the *paraboloid*, or *parabolic conoid*, and the *hyperboloid*.

OBLATE SPHEROID.



An *oblate spheroid* is a solid generated by the revolution of an ellipse about its *minor axis*.

An oblate spheroid is equal to two-thirds of a cylinder, whose diameter is equal to the greater or equatorial diameter of the spheroid, and its length equal to the polar diameter, or polar axis. Therefore,

To find the solidity of an oblate spheroid :—

Multiply the square of the equatorial diameter by the minor or polar diameter, and the product by .5236, (that is, by two-thirds of .7854,) *and the result will be the solidity.*

EXAMPLES.

1. What is the solidity of an oblate spheroid, whose polar axis is 15, and equatorial axis 25 ? *Ans.* 4908.75.

2. What is the content of an oblate spheroid in feet and bushels, its greater axis being 4 feet, and its less axis 3 feet ?

Answers, 25.1328 feet ; and 20.2 bushels, nearly.

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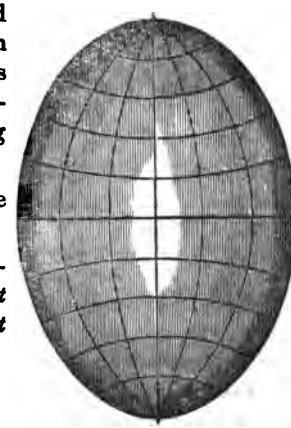
To *gauge* an oblate spheroid :—Place two-thirds of the length of the polar axis over the gauge points of the cylinder, and over the equatorial diameter in inches found on D, will be found the contents on the line C. See ¶ 36, page 119.

3. What is the content of an oblate spheroid in ale and wine gallons, its diameters being 24 and 36 inches?

Answers, 70.5024 wine, and 57.7517 ale gallons.

¶ 49. PROLATE SPHEROID.

A prolate spheroid is a solid generated by the revolution of an ellipse about its major axis. Its solidity is equal to that of two-thirds of its least circumscribing cylinder. Therefore,



To find the solidity of a prolate spheroid :—

Multiply the square of the minor by the major axis, and that product by .5236, and the result will be the solidity.

EXAMPLES.

1. Find the solidity of a prolate spheroid, whose major axis is 7 and its minor axis 5. *Ans.* 91.63.

2. What is the solidity of a prolate spheroid, whose axes are 18 and 14? *Ans.* 1847.26.

3. What is the content of a prolate spheroid in feet, bushels, and wine and ale gallons, its axes being 36 and 24 inches?

Answers, 6.3 feet; 5.06 bushels; and 47 wine, and 38.5 ale gallons.

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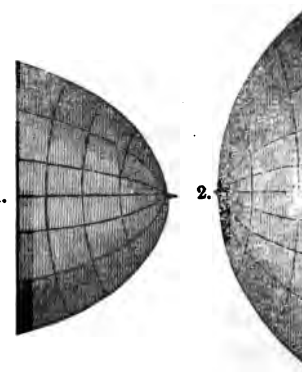
To *gauge* a prolate spheroid:—Place two-thirds of the major axis over the gauge points for the cylinder, and over the minor axis found on D, will be found the contents on C.

¶ 50. SEGMENTS OF SPHEROIDS.

The segment of a spheroid is a portion cut off by a plane perpendicular to one of its axes.



When the plane is perpendicular to the fixed axis, (as represented in figures 1 and 2,) the base is a circle; but when the plane is parallel to the fixed axis, the segment has an elliptical base. The former is called a *circular segment*, and the latter an *elliptical segment*. Thus, a segment cut from a prolate spheroid by a plane perpendicular to its major axis, (as represented by figure 1,) is a circular segment; and one cut from an oblate spheroid by a plane perpendicular to its minor axis, (as represented by figure 2,) is likewise a circular segment; but when the segment is cut off by a plane perpendicular to the other axes, the base is an ellipse, and the segment is said to be elliptical.

The minor axis of the oblate spheroid is called its *polar* axis; and the major axis of the prolate spheroid is its *polar* axis.

To find the solidity of a circular segment of a spheroid:—

Multiply the difference between three times the polar axis and twice the height of the segment, by the square of the height, and the product by .5236 ; ***then say:—As the square of the polar***

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axis is to the square of the equatorial axis, so is the last product to the solidity of the segment.

To find the solidity of an elliptical segment of a spheroid:—

Multiply the difference between three times the equatorial axis and twice the height of the segment, by the square of the height, and that product by .5236; then say:—As the equatorial axis is to the polar axis, so is the last product to the solidity of the segment.

EXAMPLES.

1. The axes of an oblate spheroid are 50 and 30, and the height of a circular segment is 6; what is the solidity?

Ans. 4084.08.

2. What is the solidity of a circular segment of a prolate spheroid, the axes being 40 and 24, and the height 4?

Ans. 337.7848.

3. The axes of an oblate spheroid are 25 and 15, and the height of a circular segment of it is 3; what is the solidity of the segment? *Ans.* 510.51.

4. What is the solidity of an elliptic segment of an oblate spheroid, whose height is 10, the axes of the spheroid being 100 and 60? *Ans.* 8796.48.

5. Find the solidity of an elliptic segment of a prolate spheroid, whose axes are 25 and 15, and the height of the segment 3. *Ans.* 306.306.

¶ 51. FRUSTUMS OF SPHEROIDS.

The *middle zone* or *frustum* of a spheroid is a portion of it contained between two parallel planes at equal distances from the centre, and perpendicular to one of the axes.

To find the solidity of the middle frustum of a spheroid, the ends being *circular*:—

1. *To twice the square of the middle diameter, add the square*

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of the diameter of one end; multiply this sum by the length of the frustum, and the product by .2618, and the result will be the solidity.

When the ends of the frustum are *elliptical*:—

2. *To double the product of the axes of the middle section, add the product of the axes of one end; multiply this sum by the length of the frustum, and the product by .2618, and the result will be the solidity.*

EXAMPLES.

1. What is the solidity of a circular middle frustum of an oblate spheroid, the middle diameter being 25, the end diameters 20, and the length 9? *Ans.* 3887.73.

2. Find the solidity of an elliptic middle frustum of an oblate spheroid, the axes of the middle section being 25 and 15, and those of the ends 15 and 9, and the length 20.

Ans. 4633.86.

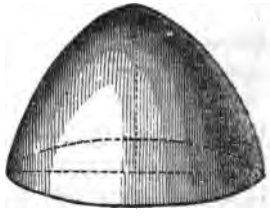
3. Find the solidity of a circular middle frustum of a spheroid, the middle diameter being 100, and those of the ends 80, and the length 36. *Ans.* 248814.72.

4. What is the solidity of an elliptic middle frustum of a spheroid, the axes of the middle section being 50 and 30, and those of the ends 30 and 18, and the length 40?

Ans. 37070.88.

¶ 52. PARABOLOID, OR PARABOLIC CONOID.

The *paraboloid* is a solid generated by the revolution of a parabola about its axis, which remains fixed. See ¶ 29. The paraboloid is often called the *parabolic conoid*, from its resemblance to the cone.



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A *frustum* of a paraboloid is a portion of it contained between two planes perpendicular to its axis.

The solidity of a paraboloid is equal to that of a cylinder of the same base and half the height. Therefore,

To find the solidity of a paraboloid:—

Multiply the area of the base by half the altitude. Or,

Multiply the square of the diameter of the base by the altitude, and that product by .3927, and the result will be the solidity.

EXAMPLES.

1. Find the solidity of a paraboloid, whose altitude is 21, and the diameter of its base 12. *Ans.* 1187.525.

2. What is the capacity of a vessel or receiver, its form being that of a paraboloid, whose base is 20 inches, and its altitude 28 inches, in ale and wine gallons?

Answers, 19.014 wine, and 15.632 ale gallons.

To gauge a paraboloid:—Place half the altitude over the guage points for the cylinder, and over the diameter of the base found on D, will be found the contents on the line C.

To find the solidity of a frustum of a paraboloid:—

Multiply the sum of the squares of the diameters of the two ends by .3927, and the product will be the mean area, which being multiplied by the altitude will give the solidity.

3. What is the solidity of the frustum of a paraboloid, whose altitude is 22.5, and the diameters of the ends 20 and 40? *Ans.* 17671.5.

4. Find the solidity of a frustum of a paraboloid, the diameters of the ends being 29 and 15, and its altitude 18.

Ans. 7535.127.

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¶ 53. HYPERBOLOID.

An *hyperboloid* is a solid generated by the revolution of an hyperbola about its axis, which remains fixed. See ¶ 30.

The hyperboloid is also called the *hyperbolic conoid*. A *frustum* of an hyperboloid is a portion of it contained between two planes perpendicular to the axis.

To find the solidity of an hyperboloid :—

To the square of the radius of the base add the square of the diameter of a section midway between the base and vertex; multiply this sum by the altitude, and the product by .5236.

To find the solidity of a frustum of an hyperboloid :—

Add together the squares of the radii of the two ends, and the square of the middle diameter between them; multiply the sum by the altitude, and this product by .5236.

EXAMPLES.

1. Find the solidity of an hyperboloid, the altitude of which is 25, the radius of the base 26, and the middle diameter 34.

Ans. 23980.9.

2. What is the solidity of an hyperboloid, the radius of the base being 6, the diameter of the middle section 10, and the altitude 20 ?

Ans. 1424.19.

3. What is the solidity of the frustum of an hyperboloid, the diameters of the ends being 6 and 10, the middle diameter $8\frac{1}{2}$, and the height 12 ?

Ans. 667.59.

4. Find the solidity of a frustum of an hyperboloid, the diameters of the ends being 3 and 5, the middle diameter 4.25, and its height 8.

Ans. 111.26.

5. What is the solidity of an hyperbolic frustum, the end diameters being 12 and 20, the middle 17, and height 18 ?

Ans. 4005.46.

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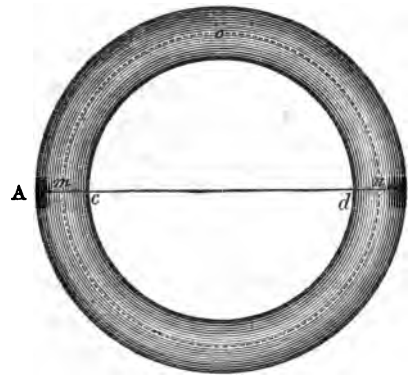
¶ 54. CYLINDRIC RINGS.

A *cylindric ring* is a solid generated by the revolution of a sphere through a circular orbit; or, it is a cylinder bent into the form of a circle. The *interior diameter* of the ring is a line passing through its centre, and limited by the interior surface; and the *external diameter* is a line passing through its centre, and terminating at its exterior surface. Half the sum of the

CYLINDRIC RING.



two diameters is the *diameter* of the *axis* of the ring; and half the difference of the diameters is the *diameter* or *thickness* of the ring. AB is the exterior diameter, *cd* the interior diameter, *mon* the axis, and Ac the thickness of the ring.

The diameter of the axis of the ring multiplied by 3.1416 gives its *length;* and the square of its thickness multiplied by .7854 gives the area of a *cross-section* of the ring, which being multiplied by the length of its axis, gives the solidity; and said length, multiplied by the circumference of a cross-section of the ring, will give its surface, or superficial area. Or,

To find the solidity of a cylindric ring:—

Multiply the square of the thickness by the diameter of the axis, and that product by 2.467412.

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To find the surface of a cylindric ring:—

Multiply the diameter of its axis by its thickness, and that product by 9.8696.

EXAMPLES.

1. Find the solidity of a cylindric ring, its diameters being 16 and 24. *Ans.* 789.57.

2. Find the solidity of a cylindric ring, its diameters being 8 and 14 inches. *Ans.* 244.274 inches.

3. The interior diameter of a cylindric ring is 26, and its thickness 8 inches; what is its solidity?

Ans. 5369 inches.

4. What is the surface of a cylindric ring, whose thickness is 1 inch, and its inner diameter 9 inches?

Ans. 98.696 square inches.

5. Find the surface of a cylindric ring, whose diameters are 36 and 52. *Ans.* 3474.1.

6. What is the surface of a cylindric ring, whose thickness is 6 inches, and inner diameter 24 inches? *Ans.* 1776.528.

¶ 55. SPINDLES.

A *spindle* is a solid generated by the revolution of a segment of an ellipse, circle, parabola, or hyperbola, about the chord or diameter, which cuts off the segment; and said chord forms the *axis* of the spindle.

The *central distance* of a circular spindle is the distance between the centre of the circle and the centre of the spindle.—Thus, *vs* is the central distance. The length of the spindle and half its middle diameter being given, (since these are the chord and versed sine of the generating circular segment,) the radius of the circle may be found, and the area of the segment, by the rules given in paragraphs 24 and 25.

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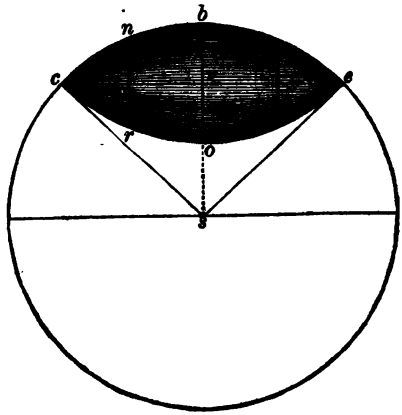
THE CIRCULAR SPINDLE.



To find the solidity of the circular spindle :—

Multiply the area of the generating segment (*cbe*) by the central distance (*vs*), and subtract the product from one-twelfth of the cube of the length of the spindle, and multiply the remainder by 6.2832. Or,

To find the solidity of the circular, the elliptic, or the hyperbolic spindle :—

To the square of the middle diameter (*bo*) *add the square of double the diameter at one-fourth the length,* (viz. *nr* ;) *multiply the sum by the length, and the product by* .1309, *and the result will be the solidity, nearly.*—(*nr* is called the *quarter diameter.*)

The *parabolic* spindle is just eight-fifteenths of its least circumscribing cylinder. Therefore,

To find the solidity of the parabolic spindle :—

Multiply the square of the middle diameter by the length of the spindle, and the product by .41888.

EXAMPLES.

1. The length of a circular spindle is 8, and its middle diameter 6 ; what is its solidity? *Ans.* 138.516.

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2. Find the solidity of an elliptic spindle, its length being 20, the middle diameter 6, and its quarter diameter 4.75.

Ans. 330.5225.

3. What is the solidity of an hyberbolic spindle, its length being 10, its middle diameter 8, and its quarter diameter 4.4 ?

Ans. 185.14496.

4. The length of a parabolic spindle is 30, and its middle diameter 17 ; what is its solidity ? *Ans.* 3631.68.

5. Find the solidity of a parabolic spindle, whose length is 18, and middle diameter 6 feet. *Ans.* 271.434.

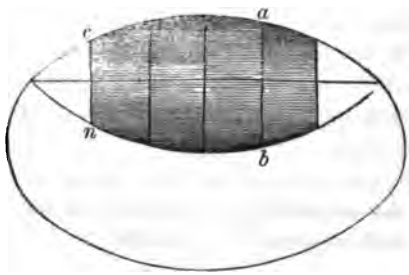
6. What is the solidity of a parabolic spindle, whose length is 50, and middle diameter 10 inches ?

Ans. 2094.42 inches.

7. The middle diameter of a parabolic spindle is 16, and its solidity 4289.33 ; what is its length ? *Ans.* 40.

An elliptical spindle is nearly four-sevenths of its least circumscribing cylinder, and its solidity may be found accordingly.

¶ 56. MIDDLE FRUSTUMS OF SPINDLES.



The figure represents the middle frustum of an elliptic spindle. The diameter, *ab*, at half the distance from the centre of the frustum to its end, is called the *quarter diameter*.

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To find the solidity of a middle frustum of an *elliptic*, a *circular*, or an *hyberbolic spindle* :—

Add together the squares of the middle and the end diameter, and the square of double the quarter diameter ; multiply the sum by the length of the frustum, and the product by .1309, *and the result will be the solidity, nearly.*

To find the solidity of a middle frustum of a parabolic spindle :—

To twice the square of the middle diameter add the square of the end diameter, and from the sum subtract four-tenths of the square of the difference between these two diameters ; multiply the remainder by the length, and the product by .2618.

EXAMPLES.

1. What is the solidity of the middle frustum of an elliptic spindle, the middle, quarter, and end diameters being 32, 30.157, and 24 inches, and the length 40 inches ?

Ans. 27424.8 cubic inches.

2. What is the solidity of the middle frustum of a circular spindle, its middle, end, and quarter diameters being 16, 15.0788, and 12 inches, and its length 20 inches ?

Ans. 3430.707 inches.

3. Find the solidity of a middle frustum of an hyperbolic spindle, whose length is 14, its middle diameter 12, the quarter diameter 11.705, and end diameter 10.8.

Ans. 1481.96516.

4. What is the solidity of a middle frustum of a parabolic spindle, the middle and end diameters being 16 and 12, and its length 30 ? *Ans.* 5101.958.

5. What is the solidity of a middle frustum of a parabolic spindle, whose length is 18, and diameters 18 and 10 inches ?

Ans. 3404.235 inches.

* Casks usually approach the form of the frustum of a parabolic spindle more nearly than any other solid. See pp. 165 and 166.

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¶ 57. IRREGULAR SOLIDS.

To find the solidity of an irregular solid of an oblong form:—

Find the areas of several equidistant sections perpendicular to the line which measures the length of the solid, and divide the sum of the areas by the number of equidistant sections, and multiply the quotient by the length. Or:—Multiply the half sum of the areas of each of the two proximate sections by the distance between them, and the sum of the several products will be the solidity.

When the solid is not great, and is very irregular, immerse it in water in some vessel of a regular form; then take out the body, and measure the capacity of that portion of the vessel which is contained between two positions of the surface of the water before and after the body was removed, and the result will be the solidity of the solid.

EXAMPLES.

1. Find the area of an oblong solid, whose length is 100 feet, and the areas of five equidistant sections, 50, 55, 70, 80, and 82 square feet. *Ans.* 6740 cubic feet.

2. What is the solidity of an oak tree of irregular form, the lengths of four portions of it being respectively 8, 5, 6, and 7 feet, and the areas of the middle sections of each part 10, 8, 7, and 5 square feet? *Ans.* 197 cubic feet.

3. An irregular mass of copper ore being immersed in water in a cylinder, whose diameter is 10 inches, the water is found to rise 6 inches higher when the solid is introduced; its solidity is required. *Ans.* 471.24 cubic inches.

4. An irregular mass of California gold being immersed in a square vessel 12 inches on a side, the water in the vessel rises 8 inches; its volume is required. *Ans.* 96 cubic inches.

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¶ 58. GAUGING.

Gauging is the art of measuring the dimensions, and computing the capacity of any vessel, or any portion of it.

When the capacity of a vessel is known in cubic inches, the number of *wine, ale,* and English *imperial gallons* may be found by dividing the capacity by 231, by 282, and by 277.274. See table IV. ¶ 12.

The divisors for circular areas, or for cylindrical vessels, are found by dividing the number of cubic inches in the measure of capacity by .785398, or .7854; *and the square roots of the divisors are the* GUAGE POINTS *on the line D.* Thus 231, 282, and 277.274 being divided by .7854, the quotients are 294.118, and 359.05, and 353.04; and the gauge points are 17.15, and 18.95, and 18.79.

In like manner the gauge points may be found for bushels, barrels, &c.

Thus, if we multiply the square of the diameter of a cylinder by its length or altitude, the dimensions being taken in inches, and divide the product by the *circular* divisors, as 294.118, 359.05, &c., the quotients will be the number of gallons, bushels, barrels, &c., which the vessel contains.

If the circular divisors are *increased* in the ratio of 2 to 3, the results are *spherical divisors;* and the square roots of the spherical divisors are the *spherical gauge points.*—For *conical* vessels the *divisors* are three times those for cylinders; and the *gauge points* are the square roots of the divisors.

The following table exhibits the *divisors* and *gauge points* for vessels of various forms, *when their dimensions are taken in inches.*

TABLE OF DIVISORS AND GAUGE POINTS.

FOR A PARABOLIC SPINDLE.

Measures.	Divisors.	Gauge Points.
Wine gallons.....	551.471	23.48
Ale gallons	673.2186	25.945

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FOR POLYGONAL VESSELS.

Measures.	Divisors.	Gauge Points.
Pentagonal Base.		
Wine gallons	134.3	11.6
Ale gallons	163.908	12.802
Imperial gallons	161.161	12.69
Bushels	1249.9	35.353
Imperial bushels	1289.288	35.91
Hexagonal Base.		
Wine gallons	88.9123	9.429
Ale gallons	108.538	10.42
Imperial gallons	106.723	10.33
Bushels	827.79	28.77
Imperial bushels	853.782	29.22
Heptagonal Base.		
Wine gallons	63.5679	7.973
Ale gallons	77.6023	8.809
Imperial gallons	76.303	8.73
Octagonal Base.		
Wine gallons	47.8414	6.916
Ale gallons	58.404	7.642
Imperial gallons	57.425	7.58
Bushels	445.3666	21.103
Imperial bushels	459.403	21.43

FOR CONICAL VESSELS.

Measures.	Divisors.	Gauge Points.
Wine gallons	882.354	29.704
Ale gallons	1077 15	32.82
Imperial gallons	1059.11	32.54
Bushels	8214	90.63
Imperial bushels	8472.87	92.049

FOR SPHERICAL VESSELS.

Measures.	Divisors.	Gauge Points.
Wine gallons	441.177	21.004
Ale gallons	538.575	23.2
Imperial gallons	529.554	23.1
Bushels	4107	64.08
Imperial bushels	4236.434	65.09

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FOR VESSELS WITH SQUARE BASES.

Measures.	Divisors.	Gauge Points.
Wine gallon	231	15.2
Ale gallon	282	16.79
English imperial gallon	277.274	16.65
Bushel	2150.42	46.372
English imperial bushel	2218.192	47.1
A pound of tallow	31.4	5.603
A pound of hard soap	27.14	5.209
A pound of dry starch	40.03	6.35

FOR CYLINDRIC VESSELS.

Measures.	Divisors.	Gauge Points.
Wine gallon	294.118	17.15
Ale gallon	359.05	18.95
English imperial gallon	353.04	18.79
Bushel	2738	52.32
English imperial bushel	2824.29	53.14
A pound of tallow	39.98	6.32
A pound of hard soap	35.65	5.97
A pound of dry starch	51.3	7.16

FOR AN EQUILATERAL TRIANGULAR PRISM.

Measures.	Divisors.	Gauge Points.
Wine gallons	533.474	23.094
Ale gallons	651.251	25.52
Imperial gallons	640.337	25.305

To gauge vessels having rectangular bases by the sliding rule:—Find the side of an equal square, (as directed under example 9, ¶ 15,) and then use the gauge points for vessels with square bases.

To gauge vessels having elliptical bases:—Find the diameter of an equal circle, (as directed under example 1, ¶ 28,) and then use the gauge points for cylindric vessels.

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TABLE OF GAUGE POINTS,

When the length or altitude is taken in FEET.

FOR SQUARE BASES.

Measures.	Gauge Points.
Bushels,	13.387
Bushel of 40 quarts,	14.996
" of 38 quarts,	14.59
Wine barrels,	24.625
Ale barrels,	29.08

FOR PENTAGONAL.

Bushels,	10.205
" for HEXAGONAL,	8.305
" for HEPTAGONAL,	7.02
" for OCTAGONAL,	6.092

FOR CYLINDRIC VESSELS.

Bushels,	15.11
Bushels of 40 quarts,	16.62
" of 38 quarts,	16.89
Wine barrels,	27.78
Ale barrels,	32.75
Wine hogshead,	39.29
Ale hogshead of 110 gallons,	57.37
Ale " of 54 gallons,	40.20
Bushels *in a sphere*,	18.50
" *in a cone*,	26.16
" for *Dodecahedron*,	7.217

See Gauge Points for the Equilateral Triangular Prism, ¶ 35; and for the Regular Solids, ¶ 39.

EXAMPLES.

1. Find the content, in pounds, of hard soap, of an oblong form, its length being 201, its breadth 60 inches, and thickness 1 inch. *Ans.* 444.36 pounds.

2. Find the content, in imperial gallons and bushels, of a vessel with a square bottom, each side being 30 inches, and its depth 40 inches. *Ans.* 129.83 and 16.229.

3. What is the content, in bushels and imperial bushels, of a cylindric vessel, whose diameter is 48 inches and depth 64 inches? *Ans.* 53.85, and 52.2.

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4. Find the content, in wine, ale, and imperial gallons, of a conical vessel, the diameter of the base being 27 inches, and its height 60 inches. *Ans.* 49.2; 40.2; and 41.3 gallons.

5. What is the content of a vessel, its form being that of a regular hexagonal pyramid, one side of the base being 40 inches, and its height 72 inches, in bushels and imperial bushels? *Ans.* 46.3 and 44.98.

In the above example, place one-third of the altitude over the gauge points for bushels and imperial bushels in the hexagonal prism, viz. 28.77 and 29.22, and over the side found on D, will be found the contents on the line C.

6. What is the capacity of a hollow sphere in bushels, its inner diameter being 72 inches? *Ans.* 91, nearly.

To gauge or find the capacity of stills, brewing-vessels, &c.:—

Divide the vessel into small portions by planes parallel to the base; find the areas of the middle sections of these portions, and multiply these areas by the corresponding depths of the portions to which they belong; the products are the cubic contents of the portions, and the sum of these products is the whole content: divide the whole content by the number corresponding to the required measure or weight, and the result is the required content.

EXAMPLE.—Find the content, in ale and wine gallons, of a flat-bottomed copper, the mean diameters at the middle of four portions of it being 54.4, 51.9, 49.6, and 47.3, and the depth of the respective portions, 12, 10, 10, and 10 inches.

Ans. 271.6, and 334.12.

When the bottom of a vessel is concave or convex, the content of the bottom portion may be most easily and accurately found by measuring the quantity of water required to fill it up, till the bottom is covered.

CASK GAUGING.

Casks are usually divided into four varieties:—The *first variety* is the middle frustum of a spheroid; the *second*, the

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middle frustum of a parabolic spindle; the *third*, two equal frustums of a paraboloid united at their bases; and the *fourth*, two equal conic frustums united at their bases.

When casks are much curved, they are considered to belong to the *first variety;* when less curved, to the *second;* when still less, to the *third;* and when the staves are straight from the bung to the head, to the *fourth variety.*

1. To find the content of a cask of the *first variety*:—

To twice the square of the bung diameter, add the square of the head diameter; multiply the sum by the length of the cask, and divide the product by 882.354 *for wine, and by* 1077.15 *for ale, and by* 1059.11 *for imperial gallons*:—That is, divide by the divisors for conical vessels, and the quotient is the content.

2. To find the content of a cask of the *second variety*:—

To twice the square of the bung diameter, add the square of the head diameter, and from the sum subtract four-tenths of the square of the difference of these diameters; multiply the remainder by the length, and divide the product by the divisors for conical vessels, and the quotient will be the content.

3. To find the content of a cask of the *third variety*:—

Add the square of the bung diameter to that of the head diameter; multiply the sum by the length, and divide the product by 588.236 *for wine, and by* 718.106 *for ale, and by* 706.0724 *for imperial gallons.*

4. To find the content of a cask of the *fourth variety*:—

Add together the product of the bung and head diameters, and their squares; multiply the sum by the length, and divide the product by the divisors for conical vessels, and the quotient will be the content.

EXAMPLES.

1. What is the content of a cask of the *first variety*, whose bung diameter is 32 and its head diameter 24 inches, and its length 40 inches, in wine, ale, and imperial gallons?

Ans. 118.95; 97.44; and 99.1.

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To gauge a cask of the *first variety* by the sliding rule:—

Add seven-tenths (or .7) *of the difference between the head and bung diameters to the head diameter, and the sum will be the* MEAN DIAMETER *of the cask, nearly; then place the length of the cask over the gauge points for cylindric vessels, and over the mean diameter found on D will be found the contents on C.* Thus, in the above example, the *mean diameter* is found to be 29.6 inches; then having placed the length, 40 inches, over 17.15 on D, over 29.6 on D will be found 118.95 wine gallons; and placing the length over 18.95, over 29.6 you will find 97.44 ale gallons; and placing the length over 18.79, over 29.6 you will find 99.1 gallons imperial measure.

2. What is the content of a cask of the *first* variety, whose bung and head diameters are 24 and 20 inches, and length 30 inches, in wine, ale, and imperial gallons?

Ans. 52.77; 43.22; and 43.96.

3. Find the content of a cask of the *second* variety, in wine, ale, and imperial gallons, the dimensions being the same as in example 1. *Ans.* 117.78; 96.49; and 98.1.

To find the *mean diameter* of a cask of the *second variety, add* .68 *of the difference between the diameters to the head diameter.*

4. Find the content of a cask of the *second* variety, in wine, ale, and imperial gallons, its bung and end diameters being 48 and 36 inches, and length 60 inches.

Ans. 398; 325.8; and 331.2.

5. Find the content of a cask of the *second* variety, in wine, ale, and imperial gallons, the bung and head diameters being 36 and 20, and its length 40 inches.

Ans. 131.03; 107.34; and 109.12.

When the difference between the diameters is great in proportion to the length of the cask, as in the last example, the *mean diameter* will be found more nearly by adding .69 of the difference between the diameters to the head diameter.

The *mean diameter* of any cask of the *first* and *second varieties* may be found very nearly by the following rule:—*Divide*

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the square of the difference between the head and bung diameters by the sum of the head and twice the bung diameter: for casks of the first variety, add one-third of the quotient; and for casks of the second variety, four-thirtieths of the quotient to two-thirds of the difference between the head and bung diameters, and the sum will be the MEAN *diameter.*

6. Find the content of a cask of the *third* variety, in wine, ale, and imperial gallons, the bung and head diameters being 30 and 24 inches, and its length 36 inches.

Ans. 90.33; 74; and 75.25.

The factor for reducing casks of the *third* variety to a *cylinder* is .54; therefore, add .54 of the difference between the head and bung diameters to the head diameter, and the sum will be the *mean diameter;* then proceed as directed under example 1. Or, to find the mean diameter of any cask which is but little curved:—To the head diameter add half the difference of the diameters; and to this sum add the quotient obtained by dividing one-fourth part of the square of the difference of the diameters, by their sum.

7. What is the *mean* diameter of a cask of the *third* variety, and what is its content in wine, ale, and imperial gallons, its diameters being 29 and 15 inches, and its length 24 inches?

Ans. 23.11; 43.4; 35.65; and 36.2.

8. Find the content of a cask of the *fourth* variety, in wine, ale, and imperial gallons, the bung diameter being 32, the end diameter 18, and the length 38 inches.

Ans. 83.34; 68.8; and 69.4.

To find the *mean diameter* of a cask of the *fourth* variety:—

To the head diameter add half the difference of the diameters; and to this sum add one-twelfth of the quotient obtained by dividing the square of said difference by the sum of the bung and head diameters.

9. Find the *mean diameter* and content of a cask of the *fourth* variety in wine, ale, and imperial gallons, its bung and

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head diameters being 40 and 20 inches, and its length 50 inches.

Ans. 30.555 = *mean diam.*; 158.7 wine; 130.1 ale; 132.2 imp. gallons.

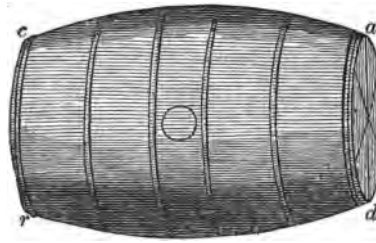
The content of any cask may be found very nearly by the general rule for the middle frustum of the spindles. See ¶ 56.

DIMENSIONS OF CASKS.

To find the dimensions of a cask:—

Take the measure of the head diameters close to the *outside*, and for small casks add three-tenths of an inch, and for casks

CASK.



of thirty or forty gallons, add four-tenths, and for larger casks add five or six tenths of an inch, to half the sum of the head diameters for the *mean* head diameter on the *inside*. Then measure the bung diameter with a rule or rod, and *observe* whether the stave opposite the bung is thicker or thinner than the rest, by moving the rod to and fro; and if it differ from the others, make the necessary allowance. Then measure the length of the cask between the outside of the heads with a rule, (or with callipers made for the purpose,) and allow for the thickness of both heads,—for casks of twenty, thirty, or forty gallons, one inch; and for larger casks allow 1.5; and for casks of more than 110 gallons allow two inches.

Having taken the dimensions of a cask, as directed above, To find its capacity in ale and wine gallons, if it be of a medium curvature:—Add the square of the head diameter to twice

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the square of the bung diameter; and from the sum subtract four-tenths of the square of the difference of the diameters; multiply the remainder by one-third of the length, and the product by .0034 for wine, by .0028 for ale, and by .002833 for imperial gallons.

ULLAGE OF CASKS.

When a cask is not full, the whole capacity is divided into two portions, one part being occupied with liquor and the other empty; either of which is called the *ullage*

To find the *ullage* of the filled part of a lying cask :—

Divide the number of *wet* inches by the bung diameter, and if the quotient is less than .5, subtract from it one-fourth of what it wants of .5; but when the quotient exceeds .5, add one-fourth of that excess to it; then, if the remainder in the former case, or the sum in the latter, be multiplied by the content of the whole cask, the product will be the ullage of the part filled.

To find the *ullage* of a standing cask :—

Divide the number of *wet* inches by the length of the cask; then if the quotient is less than .5, subtract from it one-tenth part of what it wants of .5; but if the quotient is greater than .5, add to it one-tenth of the excess above .5; then multiply the remainder in the former case, or the sum in the latter, by the content of the cask, and the product will be the ullage of the part filled.

EXAMPLES.

1. How many gallons does a lying cask contain, whose whole capacity is 40 wine gallons, the bung diameter being 20 inches, and the number of wet inches under the bung 8 ?

Ans. 15 gallons.

2. The content of a lying cask is 98 gallons, the bung diameter 32, and wet inches 10; required the ullage of the part filled. *Ans.* 26.03 gallons.

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3. The content of a lying cask is 90, its bung diameter 36, and wet inches 27; find the ullage of the part filled.

Ans. 73.1.

4. The content of a standing cask is 105 wine gallons, the length 45 inches, and the wet inches 25; what is the ullage of the part filled, in wine, ale, and imperial gallons?

Ans. 58.8; 48.166; and 49 gallons.

To find the length of a cask, its content in gallons and its head and bung diameters being given:—

Place the *mean diameter* of the cask on D, under the number of gallons on C; then over the respective gauge points on D will be found the length of the cask on C.

EXAMPLES.

1. What is the length of a cask which will hold 40 wine gallons, the mean diameter being 21 inches?

Ans. 32.5 inches.

2. The bung and head diameters of a cask of the *fourth* variety are 30 and 24 inches, and its content 60 ale gallons; its length is required. *Ans.* 29.5 inches.

¶ 59. TONNAGE OF SHIPS.

By a law of congress, the tonnage of ships is to be computed in the following manner:

"*If the vessel be double-decked, take the length thereof from the fore part of the main stern to the after part of the sternpost above the upper deck; the breadth thereof to be taken at the broadest part above the main wall, half of which breadth shall be accounted the depth of such vessel; then deduct from the length three-fifths of the breadth; then multiply the remainder by the breadth, and this product by the depth; divide the last product by 95, and the quotient will be the true tonnage of such vessel.*"

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"*If the vessel be single-decked, take the length and breadth as above directed, in a double-decked vessel; and deduct from the length three-fifths of the breadth; and, taking the depth from the under side of the deck-plank to the ceiling in the hold, multiply and divide as above directed, and the quotient will be the true content or tonnage of such vessel.*"

EXAMPLE.—What is the tonnage of a double-decked vessel, whose length is 80 feet, and breadth 24 feet?

Ans. 198.871 tons.

Ship carpenters usually compute the tonnage of a vessel by the following rule:—

Multiply the length of the keel by the breadth of the main beam, and the product by the depth of the hold, and divide the last product by 95, *and the quotient is the number of tons.*—In double-decked vessels, half the breadth of the main beam is taken for the depth of the hold.

¶ 60. COAL-PIT.

A coal-pit, well built, is very nearly equal to $\frac{2}{3}\frac{6}{3}$ of a cylinder, whose diameter is equal to that of the base of the pit, and length equal to the perpendicular height of the pit. Therefore, To find the solidity:—*Multiply the square of the diameter of the base by the height of the pit, and the product by* .32413.

To gauge a coal-pit by the sliding rule:—

Place the height of the pit in feet over 1.756 on D, then over the diameter of the base found on D, will be found its solidity in feet on C. Or:—Place the height in feet over 19.87 on D, (calling the number of feet in height so many cords,) and over the diameter in feet found on D, will be found its contents in cords on C.

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EXAMPLES.

1. How many cubic feet are there in a coal-pit, whose diameter is 12 feet, and height 8 feet? *Ans.* 373.4 feet.

2. How many cords of wood in a coal-pit, whose height is 10, and the diameter of its base 15 feet? *Ans.* 5.68 cords.

3. How many cords of wood in a coal-pit 12 feet in height, and 16 in diameter at the base? *Ans.* 7.78 cords.

¶ 61. GRINDSTONES.

Grindstones, in the form of cylinders, are sold and bought by the stone, 24 inches in diameter and 4 inches in thickness being called one stone.

To find how many stones of the above dimensions are contained in any stone:—

Multiply the square of the diameter in inches by its thickness in inches, and divide the product by 2304.

To find the number of stones in any grindstone by the sliding rule:—

Divide the thickness of the stone by 4, *and place the quotient over* 24 *on D; then over the diameter of the stone in inches found on D, will be found the number of stones which it contains on C.*

EXAMPLES.

1. How many stones 24 inches in diameter and 4 in thickness are contained in a stone 36 inches in diameter and 8 inches thick? *Ans.* 4.5 stones.

2. How many stones are contained in a grindstone 10 inches in thickness and 40 in diameter? *Ans.* 6.95.

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¶ 62. LEVELLING.

Levelling is the art of determining the difference between the *true* and the *apparent level* of two places, or of drawing a line a tangent to the earth's surface.

The *true level* is a curve parallel to the surface of water in a state of rest; and since the earth is very nearly round, it may be considered to be the *arc of a great circle.*

The *apparent level* is a *straight line,* which is a *tangent to the true level* at the point where the observation is made.

The circumference of the circle (see ¶ 22, page 72) represents the *true level,* and the tangent, *cl,* the *apparent level, c* being the place of observation; and *al* is the difference between the *true* and *apparent* in reference to the station *c*, at the distance *ca.*

Various instruments are used for the purpose of levelling; the more common of which are the *square,* the *plumb-line,* and the *spirit-level.*

Timber may usually be levelled with sufficient accuracy by the square; and if a *plumb-line* be attached to a good square, so that it may hang parallel with the shorter arm, when the longer arm is brought to a horizontal line, (or parallel to the apparent level,) it makes a very good instrument for levelling timber; and it may be used for many other purposes when great accuracy is not required.

The *spirit-level* consists of a glass tube nearly filled with spirit of wine, and enclosed in a brass tube, except the upper part. It is so formed, that when the *air-bubble* is at the middle point, the instrument indicates the *apparent level;* and by setting up rods, divided into feet and parts of a foot, perpendicular to the horizon at different stations and at equal distances from the spirit-level, and observing where the line of level or horizontal line strikes the rods, the difference of level between the different stations may be obtained; and also the *true level.*

The true mean diameter of the earth is 7912.03 miles;

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and if we assume the mean diameter of the earth to be 7920 miles, (which is near the truth,) then the difference between the true and apparent level for one mile will be expressed by $\frac{1}{7920}$ of a mile, or $\frac{5280}{7920}$ of a foot, which being reduced, gives $\frac{2}{3}$ of a foot, or 8 inches, for the *difference*, or *variation* between the true and apparent level at the distance of one mile. And since said *difference* increases nearly as the square of the distance, we may find the difference between the true and apparent level, nearly, by the following rule:—

Multiply the square of the distance in miles by 8, *and the product will be the required* VARIATION *in inches;* or, *multiply the square of the distance in miles by* $\frac{2}{3}$, *and the product will be the variation in feet.*

And when the *variation* is given, to find the distance, we may reverse the rule thus:—

Divide the height, or variation, expressed in feet, by $\frac{2}{3}$, *or* .6666, *and extract the square root of the quotient for the distance in miles.*

EXAMPLES.

1. At a certain spot in a village an observation was made for the purpose of determining whether water could be brought into the village from a spring $2\frac{1}{2}$ miles distant; and it was found that the spring and place of observation were apparently on a level; the descent from the spring to the place of observation is required. *Ans.* 4 feet 2 inches—a descent sufficient to cause water to run freely.

To find the descent by the sliding rule:—

Place .666 *on C over* 1 *on D,* (or 6 on C over 3 on D,) *and over the distance in miles found on D, will be found the required variation in feet on the line C.* Thus, over 2.5 you will find 4.17, or 4 feet 2 inches, nearly; and over 6 miles you will find 24 feet for the variation; and over 10 miles, $66\frac{2}{3}$; over 15 miles, 150 feet; and over 30 miles, 600 feet; *and if you look for the variation in feet on the line C, under the variation will be found the distance in miles on the line D.* Thus, under 1 on

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C, at the middle of the slider, (calling the 1 one foot,) we find 1.225 miles; under 2 feet, 1.73; under 6 feet, 3 miles; under 10 feet, 3.87 miles; under 20 feet, 5.45 miles; under 100 feet, 12.25 miles; and under 1000 feet, 38.7 miles.

2. The top of a lighthouse is seen at sea from the surface of the water at the distance of 23 miles; what is its height?

Ans. 352 feet.

The results obtained by the above rules, though sufficiently near the truth for short distances, when great accuracy is not required, nevertheless give results which require several corrections to render them strictly accurate. The principal correction, and the only one necessary for most practical purposes, is for *refraction.*

The rays of light in moving through the atmosphere are bent downwards, so that a height just visible at a given distance when there is refraction, would be lower than one just visible at the same distance when there is no refraction. Thus, in consequence of refraction, the distance at which an object may be seen is increased one-eleventh in ordinary states of the atmosphere; and the height of an object, or the variation, is diminished nearly one-sixth. Therefore, in finding the distance, increase the result obtained by the above rule one-eleventh; and in finding the variation, from the given distance subtract one-twelfth of itself, and then proceed as directed above, and the result will be the true height, or variation, very nearly. Or, To find the distance:—*Extract the square root of nine-fifths of the height.* And to find the height:—*Take five-ninths of the square of the distance.*

To *gauge* the distance and height by the sliding rule, allowing for refraction:—Place 1 on C over 1.336 on D, (or 8 on C over 12 on D;) then under the height or variation in feet found on C, will be found the distance in miles on D; and over the distance in miles found on D, will be found the height in feet on C.

In the following *examples* an allowance is made for *refraction.*

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3. At what distance can an object, 24 feet high, be seen at sea from the surface of the water?

Ans. 6.55, or 6.6 miles, nearly.

4. From what height will the horizon be 12 miles distant?

Ans. 80, or 80.6 feet.

5. At what distance can an object, 54 feet high, be seen at sea from the surface of the water? *Ans.* 9.8 miles.

6. At what distance can the top of a lighthouse, 216 feet high, be seen from the surface of the ocean?

Ans. 19.7 miles.

7. Required the distance of the visible horizon from the top of a hill 803 feet high. *Ans.* 37.9 miles.

8. From what height will the horizon be 36 miles distant?

Ans. 726 feet.

9. From a ship's mast, at the height of 120 feet, the top of a lighthouse, 240 feet above the level of the sea, was just visible; required their distance from each other.

Ans. 35.3, or 35.6 miles.

In the last example the distance for each of the heights must be calculated separately, and their sum will be the whole distance. By the sliding rule, having *set* the slider, under 120 feet will be found 14.8, and under 240 feet 20.8 miles on D, and 20.8 added to 14.8=35.6 miles.

10. If from the summit of a mountain, 11310 feet high, the distance of the visible horizon is 142 miles; required the earth's diameter. *Ans.* 7911 miles.

11. What is the difference between the true and apparent level at the distance of $\frac{1}{4}$ of a mile? of $\frac{1}{2}$? of $\frac{3}{4}$? of $\frac{1}{8}$? of $1\frac{1}{4}$? and of $3\frac{1}{2}$ miles?

Answers in order,—0.414; 1.65; 3.3; 0.103; 10.34; and 81 inches.

12. The difference of level between four stations is taken as follows, to find the difference between the extreme stations;

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let A be 10 feet above B, B 8 feet below C, and C 12 feet above D; the difference of level between A and D is required.

Ans. 14 feet.

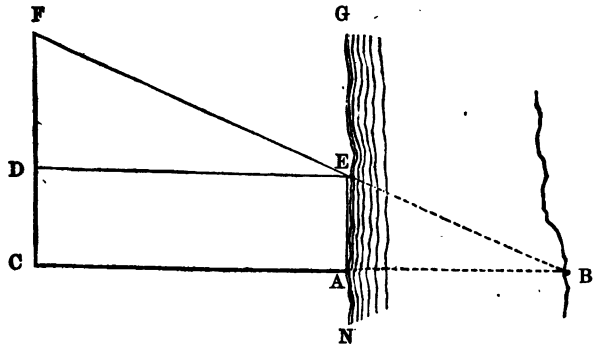
13. Let A be 12 feet above B, B 8 feet 3 inches below C, C 10 feet 11 inches above D, and D 3 feet 2 inches below E; what is the difference of level between A and E?

Ans. A is 11 feet 6 inches above E.

¶ 68. MENSURATION OF HEIGHTS AND DISTANCES.

If it be required to find the distance between two objects, one of which is inaccessible, it may be done in the following manner:—

Fig. 1.



Let GN represent the bank of a river, and B an object on the opposite bank, it being required to find the width of the stream at B, viz. AB. From the point A, measure the length of a line AE, at right angles to AB, of any convenient length, say 20 yards, and at E set up a *staff*. Then from A, at right angles to AE, measure the line AC of any convenient length, say 62 yards. Then from C, at right angles to AC, measure a

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line CF, (till you see the *staff* E and object B in the same direction,) say 45 yards. Now it is manifest, that a perpendicular from E falling on the line CF, will be equal to AC, viz. 62 yards; and that CD, the distance between said perpendicular and AC, will be equal to AE, viz. 20 yards: And if we subtract CD, or 20 yards, from CF, or 45 yards, the remainder will be FD, or 25 yards. Now, because the triangle BCF is similar to EDF, the similar sides are directly proportional. Therefore, FD is to DE as FC is to CB; or 25 is to 62 as 45 is to CB. By this statement CB is found to be equal to 111.6 yards. Therefore, AB, the required breadth of the impassable river, is 111.6—62, or 49.6 yards.

Let AE be 39.1 feet, CF 64.1, and DE 39 feet; AB is required. *Ans.* 100 feet, nearly.

To find the height of a tree or steeple, if the sun be above the horizon, and its shadow can be measured; set up a pole perpendicular to the horizon, and measure the length of its shadow. Then say:—As the length of the shadow of the pole is to its perpendicular height, so is the length of the shadow of the tree, or of any object, to its perpendicular height.

EXAMPLE.

If a pole 10 feet long cast a shadow 8 feet in length, what is the height of an object the shadow of which is 50 feet long? *Ans.* 62½ feet.

Another method is by means of two poles of unequal lengths, set up parallel to the object, so that the observer may see the top of the object over the tops of both poles.

Thus, let the length of the pole *dr* be 6 feet, that of the pole *sg* 8 feet, their distance *rg* 10 feet, and the distance *r*C of the shorter pole from the object AC, 190 feet.

Then, because the triangles *sdn* and *d*KA are similar, *dn* is to *ns* as *d*K to KA; or 10 : 8—6 : : 190 to AK. Hence, AK=38 feet; and AC=38+6=44 feet.

If the eye of the observer be placed very near the earth, then only one pole will be required. For if the eye of the observer be placed at B, and the top of the object AC be just

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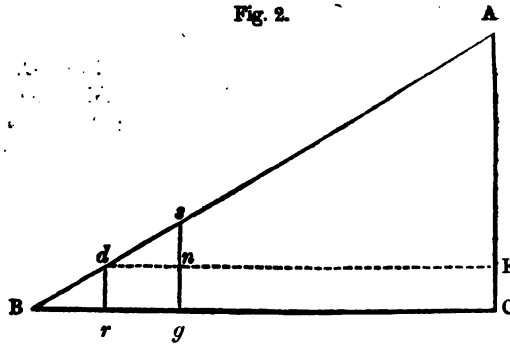
Fig. 2.



visible over the top of the pole *dr*, then B*r* : *rd* : : BC : AC. Let BC equal 100 feet, B*r* 8 feet, and *rd* 6 feet; required the height of the object AC. *Ans.* 75 feet.

Let B*g* equal 12 feet, *gs* 8 feet, and BC 90 feet; required the height of AC. *Ans.* 60 feet.

In all triangles, *the sides are to each other as the sines of the angles opposite the said sides; and any given side is to the sine of the angle opposite said side, as any other given side is to the sine of its opposite angle.* Thus, (in Fig. 2,) the sine of the angle at A is to the base BC, as the sine of the angle at B to the side AC. Or, the sine of the angle at C is to the side AB, as the sine of the angle at A is to the base BC. And the side BA is to the sine of the angle at C, as the side AC is to the sine of the angle at B. In any right-angled triangle, if either of the acute angles be given, the other may be found by subtracting the given angle from 90 degrees; and in all cases, the sum of any two angles subtracted from 180 degrees, will give the third angle. Thus, if the angle at B (Fig. 2) contain 35 degrees, the angle at A will equal 90—35, or 55 degrees; and if the angle at *a* [see the triangle ¶ 19] is 46 degrees, and the angle at *b* is 70 degrees, then the angle at *c* equals 180—(46+70,) or 64 degrees.

The natural sines corresponding to any number of degrees, and likewise the number of degrees corresponding to any natural sine, may be found in the table of natural sines, ¶ 73.

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EXAMPLES.

1. Given, the base of the right-angled triangle ABC, viz. BC 96 rods, and the angle at B (between the base and the hypotenuse) 18 degrees; required the perpendicular AC.

Ans. 31.192 rods.

The angle at A equals 90—18=72 degrees; and in the table (¶ 73) we find the sines of 72 and 18 degrees to be .951057 and .309017; therefore, as .951057 : 96 : : .309017 to AC.

2. If the perpendicular of the right-angled triangle EBA (Fig. 1) be 40 rods, and the angle at E is 70 degrees, what will be the breadth of the river AB? *Ans.* 109.89 rods.

3. If the base of a triangle is 60 feet, and the angles at the extremities of the base are 50 and 60 degrees; required the other two sides. *Ans.* 55.296 and 48.912 feet.

4. The base of a right-angled triangle is 222 feet, and the hypotenuse 423 feet; required the two acute angles.

Ans. 31° 40′ and 58° 20′.

Thus, as 423 : 1 : : 222 to the sine of the angle opposite the base, viz. 0.524822; and .524822 *minus* .515038, the sine of 31 degrees, =.009784; and .529919, the sine of 32 degrees, *minus* .515038=.014880; then, as .014880 : 60 : : 0.009784 to the number of minutes which the angle opposite the base exceeds 31 degrees; and consequently, the angle equals 31 degrees, 40 minutes, nearly; and 90—31° 40′= 58° 20′.

The *complement* of an angle is what it wants of 90 degrees, and the *supplement* of any angle is what it wants of 180 degrees. Thus, the supplement of 106 degrees is (180—106), or 74 degrees; and the sine of any obtuse angle is the same as the sine of its supplement.

5. The angle opposite to the diagonal of a trapezium is 106 degrees, (see ¶ 20,) and the angle *cba* (viz. the angle at *b*) is 38 degrees, and the side opposite this angle, viz. *ac*, is 53 rods; the diagonal *ab* is required. *Ans.* 82.75 rods.

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As .615661 : 53 : : 0.961262 (the sine of 106, or of 74 degrees) to the side required. See ¶ 81, prob. 132.

¶ 64. IMPORTANT GEOMETRICAL RATIOS.

The *circumference* of a *circle*, or of a *sphere*, whose diameter is 1, is 3.141592654 . . .

The *area* of a *circle* whose diameter is 1, is .78539816, (or one-fourth the circumference of the circle;) and the *area* or surface of a *sphere* is four times that of a circle of the same diameter.

The *areas* of *squares* are as the squares of their sides; and the *areas* of *circles* are as the squares of their diameters; and the *areas* of *similar triangles* are as the squares of their corresponding or homologous sides; and the *areas* of all *regular polygons* are as the squares of their sides.

The *area* of the *circle* is to that of its circumscribing square as .785398 to 1; and the *area* of an *ellipse* bears the same ratio to its circumscribing rectangle.

The *triangle* is one-half and the *parabola* is two-thirds of the circumscribing rectangle.

The SOLIDITIES *of all similar solids are to each other as the cubes of their sides, or diameters. Similar cones* or *similar pyramids* are to each other as the cubes of their bases, and likewise as the cubes of their altitudes.

A *cylinder* bears the same ratio to its circumscribing square prism, that the circle does to its circumscribing square, viz. that of .7854 to 1.

A *cone* is one-third of its least circumscribing cylinder; and a *pyramid* is one-third of its least circumscribing prism of the same number of sides.

A *sphere* is .52359878 of its circumscribing cube, and two-thirds of its circumscribing cylinder.

A *spheroid* is two-thirds of the circumscribing cylinder, whose length is equal to that of the polar axis of the spheroid.

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The *paraboloid* is one-half of its circumscribing cylinder.

The *parabolic spindle* is eight-fifteenths of its circumscribing cylinder.

¶ 65. RULES FOR THE SOLUTION OF A VARIETY OF DIFFICULT GEOMETRICAL PROBLEMS.

1. The base of a right-angled triangle and the sum of its three sides being given, to find the perpendicular:—

From the sum of the three sides subtract the base, and from the square of the remainder subtract the square of the base, and divide the LAST *by* TWICE *the* FIRST *remainder.*

EXAMPLE.—The height of a tree standing upon a plane is 80 feet; at what distance from the ground must it be broken off, that the top may strike the plane 20 feet from the stump, whilst the other end rests on the top of the stump?

Ans. 26$\frac{2}{3}$ feet.

2. Given the sum of two numbers and the difference of their squares, to find those numbers:—

Divide the difference of their squares by their sum, and the quotient will be their difference; add half of this difference to half the sum for the greater, and subtract half of this difference from half the sum for the less number.

EXAMPLE.—The sum of the sides of a rectangle is 64, and the difference between the squares of the length and breadth is 256 rods; required the length of the sides.

Ans. 20 and 12 rods.

3. Given the difference of two numbers and the difference of their squares, to find the numbers:—

Divide the difference of their squares by the difference of the numbers, and the quotient will be their sum; and half their difference, added to and subtracted from half their sum, will give the numbers.

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EXAMPLE.—The difference between the opposite corners of a rectangular field and one of its sides is 20 rods, and the difference of their squares is 2000; the diagonal and the sides, and the area are required.

Ans. The diag. = 60 rods; the sides 40 and 44.7213 rods; and the area 1788.8544 square rods.

4. Given the sum and difference of the squares of two numbers, to find the numbers:—

From the sum of their squares subtract the difference of their squares, and half the remainder will be the square of the less, which, subtracted from the sum of their squares, gives the square of the greater number.

EXAMPLE.—The sum of the squares of two sides of a rectangle are 208, and the difference of their squares 80; what are the sides? *Ans.* 8 and 12.

5. Given the sum of two numbers and the sum of their squares, to find the numbers:—

From the square of their sum subtract the sum of their squares, and deduct the remainder from the sum of their squares, and the square root of the last remainder will be the difference between the two numbers, half of which being added to and subtracted from the half sum, will give the numbers required.

EXAMPLE.—A and B have 50 guineas between them, and it is required to divide them in such a manner that the sum of the squares of the two numbers shall be 1300; how many guineas will each possess, supposing A has the greater number? *Ans.* A 30, and B 20.

6. Given the sum of the squares of two numbers, and the square of half their sum, to find the numbers:—

From the sum of their squares subtract double the square of their half sum, and the square root of half the remainder will be half their difference.

EXAMPLE.—If the sum of the squares of two numbers be 3161, and the square of half their sum 156.25, what are the numbers? *Ans.* 44 and 35.

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7. Given the difference of two numbers and the sum of their squares, to find the numbers :—

From the sum of their squares subtract the square of their difference, and add the remainder to the sum of their squares, and the square root of this sum will be the sum of the required numbers.

EXAMPLE.—A certain number of acres of land are to be divided between A and B in such a manner that A shall have 50 acres more than B, and that the sum of the squares of their respective portions shall be 12,500; how many acres had each? *Ans.* A 100, and B 50 acres.

8. Given the sum and product of two numbers, to find the numbers :—

From the square of half their sum subtract the square of their product, and the square root of the remainder will be half the difference between the numbers.

EXAMPLE.—A rectangular plot of land is 100 rods long and 80 broad; required the breadth of a gravelled walk extending round the field on two sides, and covering half its area.

Ans. 25.96876 rods.

The area of the walk is half the product of the two sides, or 4000, and the square of 90 (half the sum of the two sides) is 8100; from which deduct 4000, and the square root of the remainder, viz. 64.03124, subtracted from 90, leaves 25.96876, the breadth of the walk.

9. Given the difference between two numbers, and given two other numbers, to find four numbers such that the sum of two of them shall equal one of the given numbers, and the sum of the products of these two into the other two required numbers shall equal the other given number.

The rule in this case will be understood best by the following *examples* and solutions :—

1. Two men, A and B, purchase a farm of 600 acres for 1200 dollars, each paying 600 dollars for the farm; but when they come to divide the land between them, A agrees to pay

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three-fourths of a dollar, or 75 cents per acre more than B, for the privilege of selecting the more valuable portion of the field for himself, to which B assents; required the price which each must pay per acre, and the number of acres each must have to satisfy the conditions of the statement.

Ans. The share of A is 245.8 acres, for which he pays $2.443000463 per acre; and B's share is 354.2 acres, for which he pays $1.693000463 per acre.

SOLUTION.—$\frac{1200}{600}$, or $2, is the *mean* price per acre which each pays on *purchasing* the field. Add the square of *this mean* rate per acre to the square of the difference in price per acre paid by each on dividing the land, and extract the square root of the sum. [$2 \times 2 + \frac{3}{4} \times \frac{3}{4} = 7\frac{3}{16}$, the square root of which is 2.136000925.] To this root add the *mean* price per acre, viz. $2, and half the sum will be the *mean* price paid per acre on *dividing* the land. [2.136000925 + 2 = 4.136000925, which divided by 2, gives the *mean* between what each pays on dividing the land, viz. $2.0680004625.] To *this mean rate* add half the given difference in price, and from it subtract half of said difference, (viz. half of $\frac{3}{4}$, or of 75 cents,) and the sum and remainder will be the required *prices* per acre. By each of *these* divide $600, and the quotients will be the number of acres which each must share.

2. Suppose the above farm costs 1800 dollars, each paying 900 dollars of the purchase-money; required the price which each must pay per acre on dividing the land, the conditions being the same as in example first.

Ans. A pays 3.421165 dollars, and B 2.671165 dollars per acre.

3. Suppose, on dividing the land, A agrees to pay one-third more per acre than B, the conditions of the question being in other respects the same as in example first; required the price which each must pay per acre, and the number of acres which each must receive to satisfy the conditions of the agreement.

In this case, (and all similar ones,) we may reason thus. Where B pays (on dividing the land) 1 dollar, A must pay

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$1 + \frac{1}{3}$, or four-thirds; consequently, when both pay $\frac{7}{7}$, B will pay $\frac{3}{7}$ and A $\frac{4}{7}$. Therefore, $\frac{3}{7}$ and $\frac{4}{7}$ will express the relative number of acres which each must receive, B receiving $\frac{4}{7}$, and A $\frac{3}{7}$ of 600 acres, viz. $342\frac{6}{7}$, and $257\frac{1}{7}$ acres. And 600 dollars divided by $342\frac{6}{7}$, and by $257\frac{1}{7}$, will give the prices per acre, viz. \$1.75 and \$2.33$\frac{1}{3}$.

4. Suppose A pays one-fourth per acre more than B on dividing the land, the conditions in other respects being the same as in example second; required the number of acres and prices per acre.

Ans. A receives $266\frac{2}{3}$ acres, and pays \$3.375 per acre; and B receives $333\frac{1}{3}$ acres, and pays \$2.70 per acre.

10. Given the diameter of a cylinder, to divide it into *any number* of *equal parts* by means of concentric circles, or by cutting off successively equal portions from the convex surface:—

Divide the square of the diameter by the number of equal parts, and subtract the QUOTIENT *from the square of the diameter, and the square root of the* REMAINDER *will be the diameter after cutting off the first equal part. Again, subtract the same* QUOTIENT *from the* REMAINDER, *and the square root of this remainder will be the diameter after cutting off the second equal part. Then subtract the same* QUOTIENT *from the second remainder, and proceed as before. Then subtract the same* QUOTIENT *from the third remainder, and proceed as before; and so on to the last, and the square roots of the several remainders will be the diameters of the different rings, or sections.*

EXAMPLE. Suppose seven men purchase a grindstone, each paying for one-seventh part of the stone; how much must each grind off from the diameter in order to get his share, the diameter of the stone being 60 inches, and the partners grinding successively?

Ans. in order. The *first* that grinds will lessen the diameter of the stone 4.4508 inches; the *second* 4.84; the *third* 5.353; the *fourth* 6.0765; the *fifth* 7.2079; the *sixth* 9.3935; and the *seventh* finds the diameter 22.6778 inches.

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To solve the last example by the sliding rule:—

Place the number of equal parts, or shares on C, viz. 7, over the diameter on D, viz. 60 inches; then under the several shares, or equal parts, found on C, will be found the diameters of the several concentric circles which divide the stone into the required number of equal parts. Thus, under 6 will be found 55.549 inches, which subtracted from 60 gives 4.451, the number of inches which the *first* grinds from the diameter of the stone; and under 5 will be found 50.71 inches, which subtracted from 55.549, leaves 4.839 inches, the number of inches cut from the diameter of the stone by the *second* one that grinds; and thus proceed with the others.

11. Given the length of the rails, and the number in a length, to find the size or magnitude of a square piece of land, such that the number of acres, and the number of rails required to fence or enclose it, shall be equal:—

Divide the length of one rail by four times the number of rails in one length, and the square of the quotient will be the area which might be enclosed by one rail alone. Then say: As the area enclosed by one rail is to the area of one acre, so is one rail to the required number of acres, or rails. Or, *The area enclosed by one rail is to the area of one rod, or one acre, or one mile, as is one rail to the required number of rods, acres, or miles.*

EXAMPLES.

1. A square piece of ground is enclosed by a three-rail fence; the length of each rail is 15 feet, and the number of rails and the number of acres are equal; required the side of the field, the area, and number of rails.

Ans. 2,112 rods is the side, and the number of rails and acres are 27,878.4 each.

The side of a piece which may be fenced by one rail is expressed by $\frac{15}{4\times 3}$, or $\frac{5}{4}$, and its area is consequently $\frac{25}{16}$ of a

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square foot. Therefore $\frac{2}{16}$: 43560, the number of feet in an acre, : : 1 rail is to the required number of acres.

2. A gentleman who had two daughters, Eunice and Adelia, gave to each a piece of land valued at the rate of $400 per acre. The piece which he gave to Eunice was square, and (calling the diameter of a dollar one inch) the number of dollars which would enclose the piece would pay for it. The piece which he gave to Adelia was round, and the number of dollars which would encircle it would pay for it; what was the value of each piece, and the number of acres in each?

Ans. 627.264 acres = the area of the square piece, and $250,905.60 = its value: 492.652 acres = the area of the round piece, and $197,060.80 = its value.

On the principle illustrated by the first example under this proposition, one dollar would enclose one-sixteenth of a square inch, and in this example one dollar must enclose $\frac{1}{400}$ of an acre, or 15681.6 square inches. Therefore, 15681.6×16 will give the value of the square piece, viz. $250,905.60; and $\frac{250905.60}{400} = 627.264$ acres, which multiplied by 0.7853982 gives 492.652, the number of acres in the round piece of land.

12. Given the product and difference of two numbers, to find the numbers:—

To the product of the two numbers add the square of half the difference, and the square root of the sum will be the MEAN *between the two numbers, to which add half the difference for the greater, and subtract half the difference for the less.*

EXAMPLE.—The area of a rectangle is 4.5 acres, or 45 square chains, and the length exceeds the breadth by 4 chains; required the length and breadth of the rectangle.

Ans. 9 and 5 chains.

13. Given the product of two numbers, and the sum of their squares, to find the numbers:—

Find the difference between the square of their product and the square of half the sum of their squares, and extract the square

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root of this difference; to this root add half the sum of the squares, and the square root of this sum will be one of the numbers required, by which divide the product, and the quotient will be the other.

EXAMPLE.—The area of a rectangle is 48 chains, and the distance between the opposite corners, or the diagonal, is 10 chains; the sides are required. (The square of the diagonal will be the sum of the squares of the required numbers.)

Ans. 8 and 6 chains.

14. Given the heights of two objects standing on a plane, and the distance between them, to find the point in the plane equidistant from the summits of each:—

As the distance between the two objects is to the sum of their heights, so is the difference between their heights to the difference in the distance of the REQUIRED POINT *from the bases of the respective objects: therefore, multiply the sum of their altitudes by their difference, and divide the product by the distance from one of the objects to the other, and half the quotient added to and subtracted from half the distance between the objects, will give the distances of the* REQUIRED POINT *from the bases of each of the objects.*

EXAMPLES.

1. The distance between two perpendicular walls standing on a plane is 60 feet, and their altitudes are 80 and 70 feet; at what distance from the base of the lower wall must the foot of a ladder be planted, in order that its top may just reach the top of each wall? and what will be the length of the ladder?

Ans. 42.5 feet, and the length of the ladder = 81.8917 feet.

2. Two towers standing on a plane are 200 feet apart, and their altitudes 30 and 50 feet; find the point in the plane equidistant from the tops of both towers, and the length of the ladder that will reach their summits, its foot being placed at the required point.

Ans. The required point is 104 feet distant from the base of the lower tower, and the length of the ladder is 108.2405 feet.

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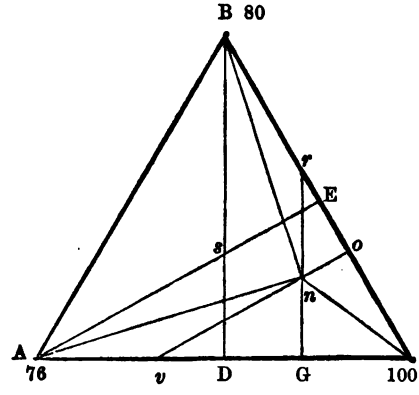
3. The side of an equilateral triangle is 100 feet, and at each of the angles a tower is erected, one 76, one 80, and one 100 feet in height; the point (in the triangle) equidistant from the summits of the three towers is required, and the length of a ladder which will reach from this point to the top of each of the towers.

Ans. The required point is 35.2872857 feet from the highest tower; and the ladder is 106.04335214 feet.



Let ABC be the equilateral triangle; the tower at A 76 feet, that at B 80, and that at C 100 feet. The apothegm, BD, of the triangle is 86.6025403784 feet; and D*s*, the distance from D to the *centre* of the triangle, is one-third of BD, =28.86751345948 feet. G, the point equidistant from the summits of the towers at A and C, is 71.12 feet from A, and 28.88 feet from C; and *o*, the point equidistant from the summits of the towers at B and C is 68 feet from B, and 32 from C. From G let G*r* be drawn at right angles to AC; and let *ov* be drawn from *o* at right angles to BC. Then, since the *required point* must necessarily lie in each of these lines, viz. G*r* and *vo*, *n* must be the required point. Now, since E*o* is 18 feet, and EC is half of AC, and *vo* is parallel to AE, A*v* must be double E*o*, or twice 18 feet, = 36 feet. Therefore *v*G=71.12−36=35.12 feet. And since the triangles AD*s* and *v*G*n* are similar, AD : D*s* : : *v*G : G*n*; that is,

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50 : 28.86751345948 : : 35.12 : G*n*. Therefore,

G*n*=20.27654145393875; and G*n* square added to the square of GC, will give the square of C*n*; to which square if we add the square of the height of the tower at C, (viz. the square of 100,) it will give the square of the length of the ladder. And if from the square of C*n* we subtract the square of C*o*, the remainder will be the square of *no*; to which if we add the square B*o*, and to this sum the square of the height of the tower at B, it will give the square of the length of the ladder the same as before; which is proof that *n* is the true point required.

15. To cut off any given portion from a triangle by a line parallel to the base:—

As the whole area of the triangle is to the square of its base, so is the area of the portion cut off to the square of its base. Or, *As the whole area is to the square of the perpendicular, so is the area of the portion cut off to the square of its perpendicular.* Or, *the whole area is to the square of either side, as is the area of the portion cut off to the square of its corresponding side; the square root of which will be the side required.*

EXAMPLE.—The sides of a triangle are 12, 18, and 20 rods; it is required to divide the triangle into two equal portions by a line parallel to the longest side. Into what two parts will the proposed line divide the side of the triangle which is 12 rods? *Ans.* 8.48527, and 3.51473 rods.

16. To cut off any portion from a triangle by a line drawn from its vertex upon the base:—

Divide the area of the portion required to be cut off by the altitude of the triangle, and the quotient will be half the length of the base of the part cut off; or half the distance from one of the angles at the base of the triangle to the point where the dividing line cuts the base.

EXAMPLES.

1. The length of one side of a triangular field is 100 rods, and the perpendicular on it from the opposite corner is 49.6

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rods; it is required to cut off a triangular portion from it by a line drawn from the same angle to this side, so that its area shall be 5 acres 16 rods: required the distance of the dividing line from the nearest angle at the base of the triangle.

Ans. 32.904 rods.

2. Cut off a portion containing 2 acres 1 rood 24 rods from a triangle, (as in the preceding example,) the length of one side being 1280 links, and the perpendicular on it, from the opposite corner, 750 links.

Ans. length of base = 320 links.

17. Given the diameter of a circle, to find the diameter of one of the three greatest equal circles which can be inscribed in it:—

Divide the given diameter by 2.1547004; *or multiply it by* 0.4641016, *and the quotient in the former, or product in the latter case, will be the required diameter.*

EXAMPLES.

1. What is the diameter of one of the three greatest circles which can be inscribed in a circle whose diameter is 100 rods?

Ans. 46.41016 rods.

2. What is the area of a circle, in which you can inscribe three equal circles, each 60 rods in diameter?

Ans. 82.0440542 acres.

18. To find the area of the triangle, or the space included between the circumferences or arcs of three equal circles which touch each other:—

Multiply the square of the diameter of one of the equal circles by 0.4330127, *and from the product subtract half the area of the circle, and the remainder will be the required area.* Or:— *Multiply the square of the diameter of one of the equal circles by* .04031362, *and the product will be the required area.*

EXAMPLES.

1. If three equal circles, whose diameters are each 46.41

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chains, touch each other, what is the area of the triangular space included between their circumferences?

Ans. 8.6831 acres.

2. A widow, who had a farm of 500 acres in the form of a circle, gave her three sons the three largest inscribed circles which could be formed in the circle of 500 acres; and to her three daughters she gave the three pieces of land included between the circumferences of the three inscribed circles and the perimeter of the 500 acres, and reserved for herself the triangular piece included between the three circles which she gave to her sons; required the number of acres possessed by each.

Ans. The widow retains 5 acres and 84.378 rods, and gives to each of her sons 107 acres and 111.22655 rods, and to each of her daughters 57 acres and 21.492 rods.

19. Given the base and altitude of any triangle to find the side of the greatest inscribed square:—

Divide the product of the base and perpendicular by their sum, and the quotient will be the side of the required square.

EXAMPLES.

1. The base of a right-angled triangle is 28, and the perpendicular 12; what is the side of the inscribed square?

Ans. 8.4.

2. The base and perpendicular of a triangle are 28 and 21; required the side of the inscribed square. *Ans.* 12.

20. Given the diameters of two wheels of unequal size, and the length of the axletree on the extremities of which they are placed, to find the diameter of the circle described by the larger wheel, when they are set rolling on a plane:—

Multiply the diameter of the larger wheel by the length of the axletree, and divide the product by half the difference of the diameters of the two wheels.

EXAMPLE.—The diameter of the greater of two wheels is 8 feet and that of the less 6 feet, and the length of the axletree

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to which they are attached is 20 feet; required the diameter of the circle described by the larger wheel, when rolling on a plane. *Ans.* 160 feet.

21. Given the diameter of a globe, to find the side of the greatest tetrahedron which can be cut from it:—

Multiply the diameter of the sphere by .8174, *and the product will be the required side.*

EXAMPLE.—I demand the side of the greatest tetrahedron that can be cut from a globe whose diameter is 22 inches.

Ans. 17.98 inches.

22. The diameter of a globe being given, to find the altitude of the greatest tetrahedron that can be cut from it:—

Multiply the diameter by .667402.

EXAMPLE.—What is the altitude of the largest tetrahedron that can be cut from a sphere 9 feet in diameter?

Ans. 6.00667 feet.

23. To find the solidity of *circular, elliptical,* or *gothic-vaulted* roofs:—

Multiply the area of one end by the length.

EXAMPLE.—Find the solidity of a semicircular vault, whose span is 40 feet, and length 40? *Ans.* 25132.8 feet.

24. To find the solidity of a dome:—

Multiply the area of the base by two-thirds of the altitude.

EXAMPLE.—What is the solidity of a spherical dome, the diameter of its base and its height being 30 feet each?

Ans. 14137.2 feet.

25. To find the length of a cord that will wind round a cylinder of a given diameter a given number of times, the distance between the spirals being known:—

To the square of the circumference of the cylinder add the square of the distance between the spirals, and the square root of the sum will be the length of the chord required to wind once round the cylinder, which multiplied by the number of times the cord passes around the cylinder, will give its length.

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EXAMPLE.—What is the length of a cord that will wind 4 times round a cylinder 7 inches in diameter, the distance between the spirals being 12 inches? *Ans.* 8.3507 feet.

When great accuracy is required, add the thickness of the cord to the diameter of the cylinder, and the sum will be the diameter described by the centre of the cord.

26. To find the lunar surface of a sphere, or the area of the part included between two meridians:—

Multiply the diameter of the sphere by the equatorial distance between two meridians: Or, *Multiply the whole surface of the sphere by the number of degrees between the two meridians, and divide the product by* 360.

EXAMPLE.—The diameter of a sphere is 40 feet, and the equatorial distance between two meridians 5 feet; required the lunar area. *Ans.* 120 feet.

A *spherical triangle* is a portion of the surface of a sphere bounded by the *arcs of three great circles.* The *spherical excess* is the *excess* of the three angles above two right angles, or 180 degrees.

27. To find the area of a spherical triangle:—

Multiply the spherical excess by one-fourth of the surface of the sphere, and divide the product by 180 *degrees.*

EXAMPLES.

1. The three angles of a spherical triangle are 60, 65, and 85 degrees, on a sphere whose diameter is 20; required the area of the triangle. *Ans.* 52.36 feet.

Two right angles are 180 degrees; consequently, the spherical excess in the above example is 30 degrees.

2. The diameter of a sphere is 50, and the angles of a spherical triangle described on it are 75 degrees, 15 minutes; 82 degrees, 12 minutes; and 35 degrees and 3 minutes; find the area of the triangle. *Ans.* 136.3.

3. If the number of square miles on the surface of the earth be 196,751,340, (which is near the true surface;) required

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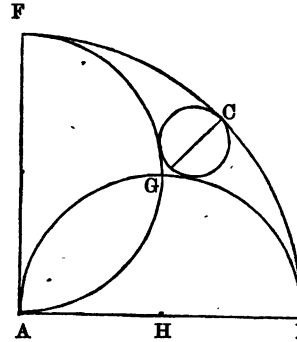
the area of a spherical triangle on its surface, the *excess* of the three angles above two right angles being 1 second.

Ans. 134.9459 acres.

28. Suppose that two right lines, AF and AB, meet each other and form a right angle at A, and that the arc of a circle, with a given radius, and its centre at A, be struck, cutting the two right lines at F and B; then, suppose that two semicircles, with one-half the given radius, and having for their centres the centres of the radii of the former arc, be struck within the said arc, to find the *diameter* of a circle described between and touching the three arcs, viz. CG:—



Extract the square root of double the square of (AH) *the radius of the smaller arc, and subtract half of this root from the radius of the larger arc; square this* REMAINDER, *and to its square add the square of half the root previously found, and from this sum subtract the square of the radius of one of the semicircles, and* RESERVE *this remainder for a* DIVIDEND; *then, to double the* REMAINDER *first found, add twice the radius of one of the semicircles* (or the radius of the quadrantal arc) *for a divisor; by which divide the* RESERVED *dividend, and the quotient will be the* RADIUS *of the circle, whose diameter is required.*

EXAMPLE.—Let the radius of the quadrantal arc be 100 rods, and that of each of the semicircles 50; the diameter of the inscribed circle CG is required. *Ans.* 25.547829 rods.

29. Given, the *hypotenuse* of a right-angled triangle and the *side* of the inscribed square, to find the base and perpendicular:—

From one-fourth of the square of the hypotenuse subtract one-half of the square of the given SIDE, *and* RESERVE *the* REMAINDER;

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then, extract the square root of the sum of the squares of the hypotenuse and given SIDE, *and multiply this root by half the given side, and subtract this product from the* RESERVED REMAINDER, *and extract the square root of the last remainder; which root subtract from half the square root of the sum of the squares of the hypotenuse and given* SIDE, *and add the remainder to half the given* SIDE, *and the sum will be one of the sides required.*

EXAMPLE.—The hypotenuse of a right-angled triangle is 35, and the side of the inscribed square 12; required the base and perpendicular. *Ans.* 21 and 28.

30. Given, the hypotenuse of a right-angled triangle, and the radius of the greatest inscribed circle, to find the base and perpendicular:—

Add the square of the radius to the square of the sum of the radius and hypotenuse, from which sum subtract the square of the hypotenuse; find the difference between this remainder and the square of half the hypotenuse, and extract the square root of the difference; which root subtract from half the hypotenuse, and to the remainder add the radius, and the sum will be one of the required sides.

EXAMPLE.—The hypotenuse of a right-angled triangle is 60 feet, and the radius of the inscribed circle 12 feet; the base and perpendicular are required. *Ans.* 48 and 36 feet.

31. Given the length of three lines extending from a certain point in an equilateral triangle to the three angles, to find the length of the side:—

Square the shortest of three given lines, and extract the square root of three-fourths of its square; to this root add half the sum of the other two sides, and from the sum subtract the square root of the difference between the two longest sides, and the remainder will be the required side very nearly.

EXAMPLE.—Let the three lines be 28, 31, and 20 rods; the side of the triangle is required. *Ans.* 45.089, nearly.

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¶ 66. CAPACITIES OF SUPERFICIES AND SOLIDS.

Of all plane figures, the *circle* is the most *capacious;* that is, a given perimeter will enclose the greatest area in a circular form.

Of quadrilateral figures, the *square* is the most *capacious:* and of *regular polygons*, having the same perimeter, that is the most capacious which has the greatest number of sides.

Of all solids, the *sphere* is the most *capacious;* that is, it comprehends the greatest solidity under a given surface. Of all *quadrilateral prisms*, the square prism is the most *capacious;* and of all prisms, the *cylinder* is the most *capacious.*

EXAMPLES.

1. The *standard bushel* measure is 18.5 inches in diameter, and 8 inches deep; and its capacity is 2150.42 cubic inches. Suppose that such a measure in seasoning becomes warped, and its form is changed to an ellipse, whose diameters are 19.5 and 17.5 inches; required its capacity.

Ans. 2144.137 cubic inches.

2. A pewter pint ale measure contains 35.25 cubic inches; what will be its capacity if it be beaten into a square form?

Ans. 28.9 cubic inches.

3. The areas of a circle and a square are each 4 inches; required the circumference of the circle and the perimeter of the square. *Ans.* 7.08983; and 8 inches.

¶ 67. THE BALANCE, OR SCALES.

The balance is a straight inflexible rod or beam, turning about a fixed point in the middle. If the point of suspension

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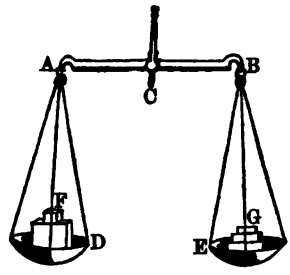
be not exactly in the middle of the beam, the balance is imperfect.



The balance serves to compare the weights of bodies, and to determine whether two or more bodies differing in bulk have the same or equal quantities of matter. A perfect balance requires the point of suspension of the beam to be in a right line with the points of suspension of the scales, and that there be as little friction as possible about the point on which the beam turns.

Such a balance will be in *equilibrio* when the scales are empty, and likewise when the scales are loaded with equal weights.

To discover a false balance :—Weigh the same body in each scale, and if the weights differ, the balance is false.

To find the *true weight* of a body by means of a false balance :—

Weigh the body, first in one scale, and then in the other ; and the square root of the product of the weights will be the true weight of the body.

EXAMPLE.—If a body is found to weigh 11 pounds in one scale and 12 pounds in the other ; what is its true weight ?

Ans. 11.489 pounds.

The weights 1, 3, 9, 27, 81, 243, and 3 times 243, and so on, will weigh any number of pounds from 1 to the sum of the weights.

¶ 68. SPECIFIC GRAVITY OF BODIES.

The *specific gravity* of a body is the ratio of its weight to the weight of the same volume of some other body assumed as a standard. Pure water being the standard commonly as-

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sumed, the specific gravity of a solid body may be regarded as the weight of a given volume of the body compared with that of an equal volume of water; and it shows how much heavier or lighter it is than the same volume of water.

Thus, if the weight of a cubic foot of any body is 10 times the weight of a cubic foot of water, its specific gravity is 10; or if the weight of a cubic foot of any body be 5 times the weight of a cubic foot of water, its specific gravity is 5, &c.

To find the specific gravity of a body heavier than water:—

Weigh the body in the air, and then in water, and divide its weight in the air by the difference between its weight in the air and in water, and the quotient will be its specific gravity.

EXAMPLE.—What is the specific gravity of a body which weighs 3 pounds in the air and 1 pound in water?

Ans. $\frac{3}{2}$, or 1.500.

To find the specific gravity of a body lighter than water:—

To the lighter body attach a heavier one that will cause it to sink, having previously ascertained the weight of the heavier body and its LOSS *in water. Then weigh the compound mass in water, and from its* LOSS, *when weighed in water, subtract the* LOSS *of the heavier body when weighed by itself, and the remainder, or difference, is the* LOSS *of the lighter body; by which* LOSS *divide its absolute weight, and the quotient will be its specific gravity.*

EXAMPLE.—What is the specific gravity of a block of wood, which weighs 15 pounds in the air, if, when attached to a piece of copper, which weighs 18 pounds in the air and 16 in water, the compound mass weighs in water only 6 pounds? *Ans.* $\frac{15}{25}$, or 0.600.

To find the specific gravity of a fluid:—

Find the loss of weight of one and the same body in water, and likewise in the fluid whose specific gravity is to be ascertained; then divide the loss in the given fluid by the loss in water, and the quotient will be the specific gravity.

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EXAMPLE.—Suppose a piece of iron, when weighed in pure water, loses 8 pounds, and when weighed in beer, loses 9 pounds, what is the specific gravity of the beer?

Ans. $\frac{9}{8}$, or 1.125.

The following table of specific gravities shows the absolute weight of each body in avoirdupois ounces. A cubic foot of distilled water weighs (at the temperature of 60°) 62½ pounds, or 1000 ounces avoirdupois; and we call its specific gravity 1, or 1.000; and the specific gravity of brick is 2, or 2.000. A cubic foot of brick will consequently weigh 2000 ounces avoirdupois.

TABLE OF SPECIFIC GRAVITIES.

Pure Water	1.000	Cast Brass	8.000
		Nickel	8.280
METALS.		" cast	7.810
Platinum	21.500	Gun Metal	8.153
Fine Gold	19.400	Manganese	8.000
Gold, cast	19.250	Soft Steel	7.850
" hammered	19.350	Steel, tempered	7.800
Standard Gold	17.724	Bar Iron	7.688
Iridium	18.700	Cast Iron	7.407
" hammered	22.900	Tin	7.320
Tungusten	17.390	" cast	7.291
Mercury, frozen	15.610	Galena	7.150
" at 60° Fahr.	13.580	Zinc	6.962
Palladium	11.800	Cinnabar	6.902
Lead	11.352	Antimony	6.702
Rhodium	11.000	Tellurium	6.115
Pure Silver	11.000	Chromium	5.900
Standard Silver	10.500	Arsenic	5.763
Bismuth	9.880		
Uranium	9.000	MISCELLANEOUS.	
Copper	7.800	Clear crystal Glass	3.000
" wire	8.900	Crown Glass	2.520
" cast	8.788	Green "	2.642
Cobalt	8.600	Flint " from 2.76 to	3.000
Cadmium	8.600	Magnetic Ore	4.000
Brass, not hammered	8.396	Ruby	4.283
" wire	8.544	Diamond	3.509

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Sapphire	4.126
Amethyst	2.750
Jasper	2.4 to 2.816
Agate	2.590
Sardonyx	2.620
Topaz	3.500
Pearl	2.684
Hornblende	3.700
Ironstone	3.281
Limestone	from 2.400 to 3.200
Granite	3.000
Red Egyptian Granite	2.654
Quartz	from 2.650 to 3.750
Porphyry	2.800
Gypsum	from 1.872 to 2.288
White Parian Marble	2.838
White Fluor Spar	3.156
White Alabaster	2.714
Spanish Chalk	2.790
Common "	1.720
Common Marble and hard Stone	2.704
Flint	2.600
Wales Slate	2.750
Rock Crystal	from 2.600 to 2.888
Basalt	2.864
Feldspar	2.650
Common Stone	2.500
Paving "	2.416
Mill "	2.484
Portland "	2.113
Grindstone	2.143
Plumbago	from 1.987 to 2.400
Clay "	1.750 to 2.160
Potash	2.012
Common Earth	1.983
Brick	from 1.6 to 2.000
Horn	1.835
Ivory	1.825
Brimstone	1.810
Sulphur	2.025
Alum	1.714
Borax	1.714
Gunpowder, solid	1.745
" well shaken	0.927
Gunpowder, loose	0.836
Sand	1.525
Loaf Sugar	1.620
Vitriol	1.841
Bone of an ox	1.659
Opium	1.336
Whalebone	1.300
Newcastle Coal	1.270
Sea Coal	1.250
Green Soft Soap	1.152
Hard Soap	1.090
Pitch	1.150
Isinglass	1.111
Rosin	1.100
Tar	1.015
Mastic	1.074
Indigo	1.009
Amber	1.040
Cheese	1.054
Beeswax	.955
Ice	.920
Butter	.942
Fat Beef	.923
Lard	.948
Tallow	.942
Camphor	.988
India Rubber	.933
Pumice Stone	.915
Charcoal	.550
Well-pressed Hay	.070
Atmospheric Air	.00125

WOODS.

Pomegranate-tree	1.351
Grape-vine, green	1.333
Lignum-vitæ	1.333
American Ebony	1.330
Heart of Oak	1.170
White Oak	.925
Mahogany	1.060
Cocoa	1.040
Olive-tree	.927
Logwood	.913
Box, French	.912
Green Ash	.851

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Beech, green	.852
" dry	.650
Citron	.726
Apple-tree	.793
Alder	.800
Maple	.755
Quince-tree	.705
Cherry-tree	.715
Lemon-tree	.703
Orange-tree	.705
Dutch Yew	.788
Elm.....from	.600 to .671
Walnut	.671
Pear	.671
Linden	.604
Larch	.544
Juniper-tree	.556
Fir	.550
Pine, dry	.453
" saturated with water	.839
Yellow Pine	.460
Cedar	.561
Willow	.585
Poplar	.383
Cork	.240

FLUIDS.

Honey	1.450
Molasses	1.290
Beer	1.250
Cider	1.018
Human Blood	1.053
Porter, Brown Stout	1.011
Strong Ale	1.035
Cider Vinegar	1.007
Water, distilled	1.000
" ocean	1.030
" Baltic Sea	1.015
" Mediterranean	1.029
" Dead Sea	1.240
Woman's Milk	1.020
Cow's Milk	1.032
" " Whey of	1.019
Liquid Turpentine	.991
Alcohol, absolute	.797
" of Commerce	.825
Proof Spirit	.920
" Brandy	.927
Port Wine	.997
White Champagne	.997
Burgundy Wine	.991
Ether, Muriatic	.729
" Nitric	.908
Naphtha..... from .700 to	.847

ACIDS.

Acetic Acid	1.062
Citric "	1.034
Fluoric "	1.060
Muriatic "	1.200
Nitric "	1.270
Sulphuric Acid	1.850

ESSENTIAL OILS.

Cinnamon	1.043
Cloves	1.036
Aniseed	.986
Sallad	.932
Wormwood	.907
Carraway-seed	.906
Lavender	.894
Turpentine	.870
Amber	.860

EXPRESSED OILS.

Spermaceti	.943
Linseed	.940
Poppy-seed	.939
Sweet Almonds	.932
Hempseed	.926
Whale and Codfish	.923
Olives	.915

One pound avoirdupois equals 7,000 grains Troy weight. The pressure of the atmosphere on a square inch is 14.7 pounds.

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ATMOSPHERIC AIR BEING THE STANDARD.

GASES.

Atmospherical Air	1.000	Sulphurous Acid	2.222
Ammoniacal Gas	0.590	Carbureted Hydrogen Gas	.972
Hydriodic-acid Gas	4.340	Prussic-acid Gas	.937
Chlorine Gas	2.500	Steam of Water at 212°	.623
Cyanogen Gas	1.805	Hydrogen Gas	.069
Nitrous-oxide Gas	1.527	Phosphureted Hydrogen	.902
Oxygen Gas	1.111		
Carbonic-oxide Gas	.972	Atmospheric Air is 816 times lighter than water,—1,000 cubic inches weighing 305 grains.	
Muriatic Acid	1.284		
Nitrous Acid	2.638		

¶ 69. ON THE WEIGHT OF BODIES.

To find the weight of a body when its cubic content, or its volume, is given:—

Multiply its cubic content, in feet, by the specific gravity of the body, (disregarding the decimal point,) and the result will be its weight in avoirdupois ounces.

EXAMPLES.

1. A cube of fine gold is two inches on a side; required its weight. *Ans.* 5 pounds 9.815 oz. avoirdupois.

2. A block of red Egyptian granite is 10 feet long, and its breadth and thickness each 20 inches; required its weight. *Ans.* 4607.5 pounds.

3. One of the stones in the walls of Baalbec was a square prism, 12 feet on a side and 63 feet long; required its weight, its specific gravity being 2.700. *Ans.* 683.44 tons.

4. What is the weight of a cylinder of cast brass, 7 feet in length and 7 inches in diameter? *Ans.* 914.82 pounds.

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5. Find the weight of a log of oak, 24 feet long, 3 broad, and 1 thick, the specific gravity being .925.

Ans. 37 cwt. 18 lbs. 8 ounces.

6. How many fir-planks, 16 feet long, 9 inches broad, and 6 inches thick, will a ship of 400 tons burden carry?

Ans. $4344\frac{8}{33}$.

To find the *gauge point* for the weight of any body:—

As the specific gravity of the body (disregarding the decimal point) is to 1728, *so is the number of ounces in the required number of pounds (as* 1 *or* 100 *pounds) to the number of cubic inches required to weigh that number of pounds; then, (having found said number of inches,) extract the square root for the gauge point, when the length of the body is taken in inches; but if the length be taken in feet, extract the square root of one-twelfth of said number of inches for the gauge point.*

What is the gauge point for red Egyptian granite, when the length in feet is placed over the gauge point, to find the number of hundreds of pounds it will weigh?

Ans. 9.317 on D.

Solution.—As 2654 : 1728 : : 1600 to the number of cubic inches required to weigh 100 pounds, viz. 1041.74, one-twelfth of which is 86.811, the square root of which is 9.317, the required gauge point.

To find the *gauge point* for the weight of any solid in the cylindric form:—

Find the number of cubic inches required to weigh 1 *or* 100 *pounds, as directed above; which number divide by* .7854, *and extract the square root of the quotient for the gauge point when the length is taken in inches, and the square root of one-twelfth of the said quotient when the length is taken in feet.*

What is the gauge point for a cylinder of cast brass, to find its weight in hundreds of pounds, the length in feet being placed over the gauge point? *Ans.* 6.12.

The *gauge point* for a millstone is 10.878; for a grindstone 12.82; for a marble cylinder 10.44; for square marble blocks

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9.25; for an octagonal prism of marble 4.21; for square iron bars 5.474; for square cast iron bars 5.59; for a wrought iron cylinder 6.20; for a cast iron cylinder 6.29; for a cast iron ball, the diameter in inches being placed over the gauge point, 2.667; for a leaden ball 2.16; for a square block of oak 15.78; for an elm log 20.917; for a cask of beer 15.318; for a cask of butter 6.1; for a cheese 5.78; for a body of well pressed hay 57.37; for a square ash stick 16.16; for a round ash log 18.69; for a square maple stick 17.47; for a round maple log 19.72; for a square stick of lignum-vitæ 13.22; for a round stick or log of lignum-vitæ 14.80; for a square walnut stick 18.6; for a round walnut log 20.91; for a square beech stick 18.82; for a round beech log 21.29; for fir when square 20.5; for a round log of fir 23.1; for apple-tree when square 17.04; for a round log of apple-tree 19.2; for water in a square form or prism, placing the *depth in inches* over the gauge point, 5.258; for water in a cylinder 5.93; for cider in a cask 5.88; for spermaceti oil in a cask 6.11; for mercury in a square prism 1.428; for mercury in a cylinder 1.610; for whale's oil in a cask 6.176; for loaf-sugar in a square form 4.131; for loaf-sugar when round 4.66; for proof spirit in a cask 6.18.

Let the student perform the following examples with the sliding rule.

7. What are the weights of the following blocks of granite, the length of each being three feet, and one being 12 inches square, one 10 inches, one 7, one 6, one 4, one 3½, one 2, and one 1 inch square?

Place the length, 3 feet, found on C, over 9.317 on D, and over 12 found on D, will be found 500 pounds on C; over 10, 346 pounds; over 7, 170 pounds; over 6, 124.5; over 4, 55.5; over 3½, 42.2; over 2, 13.75, and over 1, 3.46 pounds.

8. What are the weights of the following grindstones, the thickness of each being 5 inches, and the diameters 14, 18, 20, 30, 25, and 40 inches?

Answers in order,—59.4; 98; 121.3; 272; 190, and 486 pounds.

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A grindstone one inch thick and 12.82 inches in diameter, weighs 10 pounds; therefore, place the thickness in inches over 12.82 on D, calling the thickness so many tens, and over the diameter found on D, will be found the weight on C.

9. What is the weight of a ball of lead whose diameter is 4 inches? *Ans.* 13.714 lbs.

10. What is the weight of a cylinder of marble whose length is 10 feet and diameter 6 inches? *Ans.* 330 lbs.

11. What is the weight of an octagonal prism of marble, the side being 3 inches and its length 5 feet?

Ans. 252.5 lbs.

12. What is the weight of a cast iron cylinder, its length being 6 feet, and its diameter 2 inches? *Ans.* 60.6 lbs.

13. What is the weight of a ball of iron 4 inches in diameter? *Ans.* 9 lbs.

The solidities of balls being as the cubes of their diameters, the number of pounds in an iron ball may be found by multiplying the cube of its diameter by 9, and dividing the product by 64; and for a leaden ball multiply the cube of the diameter by 2, and divide the product by 9.

14. What is the weight of an iron ball whose diameter is 9 inches? *Ans.* 102.5 lbs.

15. What is the weight of an oak stick of timber, its length being 10 feet, and its ends 30 inches square?

Ans. 3600 lbs.

16. What is the weight of an elm log, its length being 30 feet, and diameter 13 inches? *Ans.* 1160 lbs.

17. What is the weight of a barrel of beer, its length being $2\frac{1}{2}$ feet, and mean diameter 24 inches? *Ans.* 614 lbs.

The gauge points for beer, water, and cider, in a cylinder, the length in inches being placed over the guage, are, for beer 5.31, for water 5.93, and for cider 5.88.

18. What is the weight of a mow of well-pressed hay, 6 feet square and 10 feet deep? *Ans.* 1575 lbs.

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19. What is the weight of a mow of hay 6 feet deep and 200 inches square? *Ans.* 4325 lbs.

To find the cubic contents of a body, its weight being given:—

Divide the weight of the body in ounces by its specific gravity, (disregarding the decimal point,) *and the quotient will be its cubic contents in feet.*

EXAMPLES.

1. How many cubic feet in a ton of fir? *Ans.* 63.036.

2. What is the side of a cube of fine gold, its weight being one ounce avoirdupois? *Ans.* 0.44659 of an inch.

3. What is the diameter of a sphere of platinum, its weight being 1 pound? *Ans.* 1.3492 inches.

4. Find the number of cubic feet in a ton of dry oak.
Ans. 38.75 feet.

To find the quantity of either of the ingredients in a compound consisting of two ingredients, when the specific gravities of the compound and of the ingredients are given:—

Multiply each of the three specific gravities by the difference between the other two; then, as the greatest product is to each of the other products, so is the weight of the compound to the weight of each of the ingredients.

EXAMPLES.

1. A composition weighing 56 lbs., and having a specific gravity of 8.784, consists of tin and copper of the specific gravities 7.320 and 9.000 respectively; what are the quantities of the ingredients?
Ans. 50 lbs. of copper and 6 lbs. of tin.

One statement will be all that is required; for, having found one of the ingredients, if we deduct it from their sum, or the compound, the remainder will be the other.

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2. An alloy of the specific gravity of 7.8 weighs 10 lbs., and is composed of copper and zinc of the specific gravities of 9 and 7.2; required the weight of the ingredients.

Ans. 3.85 lbs. of copper, and 6.15 lbs. of zinc.

3. An alloy of the specific gravity 7.7 consisting of copper and tin of the specific gravities 9 and 7.3, weighs 25 ounces; what is the weight of each of the ingredients?

Ans. 6.87 ounces of copper, and 18.12 ounces of tin.

¶ 70. COINS.

The average specific gravity of English gold coins is 17.500. The fineness of gold is estimated by carat grains, equivalent to 2½ dwts. Troy, pure gold being 24 carats fine. The purity of the present English gold coins is 11 parts fine gold and 1 part alloy. The sovereign, or twenty-shilling piece, contains 113.001 grains of fine gold, and 123.274 grains of standard gold. The Troy pound of standard gold is coined into 46 sovereigns and $\frac{89}{120}$ of a sovereign, or into £46 14*s.* 6*d.* The alloy in coins is reckoned of no value; and it is used simply to harden the coins, and to avoid the trouble of refining.

The commercial value of a sovereign in the United States is about $4.84.

A Troy pound, or 12 ounces of the metal of which English silver coins are made, contains 11 oz. 2 dwts. pure silver, and 18 dwts. alloy. This pound is coined into 66 shillings, each shilling containing 80.727 grains fine silver, and 87.27 grains standard silver.

The purity of American gold coins is the same as that of English gold coins; and the Troy pound of standard gold is coined into 21⅓ eagles, equal in value to $213⅓; or 9 ounces of standard gold are coined into 16 eagles, the value of which is $160; the value of one ounce being $17.77⅞. One pound of fine gold is estimated to be worth 15 pounds of fine silver.

The eagle contains 247½ grains of fine gold and 270 grains

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of standard gold, the alloy being 22½ grains. Thus 24¾ grains of fine gold (being one-tenth of the number of grains in an eagle) are worth one dollar; and if multiplied by 15 the product will be 371¼ grains, which is the number of grains of fine silver in one dollar. A Spanish milled dollar weighs 417 grains; consequently 17 of these dollars will weigh 1 pound 0 oz. 3.2548 dwts. avoirdupois, nearly.

In England, a dollar coined in the United States is valued at 4*s*. 3.68*d*. sterling.

In England the eagle is estimated to contain 246.1 grains of pure gold.

I demand the value of a cubic inch of pure gold, admitting that an eagle contains 246.1 grains. *Ans.* $199.441.

¶ 71. PILING OF BALLS AND SHELLS.

Balls and shells are usually built into piles of some regular form. The base of a pile is either an equilateral triangle, a square, or a rectangle, and is called accordingly a *triangular*, a *square*, or a *rectangular* pile. The number of balls in a side of each course diminishes by unity upwards; and the pile is said to be *complete* or *incomplete* according as it is or is not finished. The complete triangular and square piles terminate in a single ball; and the complete rectangular pile in a single row.

In the complete square and triangular piles, the number of courses is equal to the number of balls in the side of the lowest course; and in the complete rectangular pile, it is equal to the number of balls in the end of the lowest course.

1. To find the number of balls in a triangular pile:—

To the number of balls in a side of the base add 1, *and to this sum add* 1; *multiply the three numbers together; that is, find their continued product, and one-sixth of this product will be the number required.*

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2. To find the number of balls in a square pile:—

To the number of balls in a side of the base add 1, *and to twice the number in a side add* 1; *find the continued product of the three numbers, and one-sixth of this product will be the required number.*

3. To find the number of balls in a rectangular pile:—

From three times the number of balls in the length of the base, subtract the number in its breadth, less one; then find the continued product of the remainder, the breadth, and the breadth increased by 1, *and one-sixth of the product will be the number required.*

4. To find the number of balls in an incomplete pile:—

Find the number in the whole pile considered as complete, and the number in the supplementary pile, and their difference will be the number in the incomplete pile.

EXAMPLES.

1. How many balls are contained in a triangular pile consisting of 25 courses? *Ans.* 2,975.

2. How many balls are contained in a triangular pile of 40 balls in one side of the base? *Ans.* 11,480.

3. How many balls in a square pile of 20 courses? *Ans.* 2,870.

4. How many balls in a square pile having 15 balls in a side of the base? *Ans.* 1,240.

5. How many shells are contained in a rectangular pile, the number in the length and breadth of its base being 59 and 20? *Ans.* 11,060.

6. Find the number of balls in a rectangular pile of 20 courses, the number in the length of its base being 24. *Ans.* 3,710.

7. Find the number of shot in an incomplete triangular pile, the number in a side of the base and top being 40 and 20. *Ans.* 10,150.

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The whole pile considered as complete would contain 11,480; and the supplementary pile (required to complete the imperfect pile or frustum) contains 1,330, which being subtracted from 11,480 leaves 10,150 the number in the frustum.

8. Required the number of shot in an incomplete square pile of 17 courses, a side of the base containing 24. *Ans.* 4,760.

9. How many shells are contained in an incomplete rectangular pile of 12 courses, the number in the length and breadth of the base being 40 and 20 ? *Ans.* 6,146.

¶ 72. LAWS OF FALLING BODIES.

Every particle of matter in the universe has a disposition or tendency to press towards, and, if not opposed, to approach every other particle. This mutual tendency of all the particles of matter to each other is called the *attraction of gravitation.* The force of attraction is directly proportional to the masses of the attracting bodies, and inversely proportional to the squares of their distances; that is, the force of attraction diminishes in proportion to the increase of the square of the distance, and increases as the square of the distance diminishes.

Terrestrial gravity is the disposition which all heavy bodies manifest, when unsupported, to fall towards the centre of the earth, in consequence of the earth's attraction; which attraction operates in the same manner as if all its matter were condensed into a single point at its centre.

Since gravity acts with very nearly the same degree of force on a falling body during the whole time of its descent, it may be regarded as a *uniformly accelerating force.*

Terrestrial gravity acts equally on all bodies; and consequently, were it not for the buoyancy of the atmosphere, a piece of gold and a feather would fall with the same velocity.

It has been demonstrated by experiment, that any body (were it not for the resistance of the atmosphere) would fall 16.1 feet the first second, 4 times 16.1 feet in two seconds,

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9 times 16.1 in three seconds; 16 times 16.1 in four seconds, and so on; the whole space fallen through in any number of seconds, being equal to the square of the time in seconds multiplied by 16.1 feet. Now a heavy body, as a ball of iron or of lead, in falling through the atmosphere, is governed by the same law, only its velocity is slightly impeded by the resistance of the atmosphere, for which it is proper to allow one-tenth of a foot for every 16 feet. Hence,

To find the distance fallen through by any heavy body in a given time:—

Multiply the square of the time in seconds by 16, *and the result will be the distance in feet.*

EXAMPLES.

1. A man fell from a balloon, and was 25 seconds in reaching the ground; required the height of the balloon.

Ans. 10,000 feet.

2. A shell fired from a cannon, after having reached its greatest height was 20 seconds in falling to the earth; required the height which it reached above the earth.

Ans. 6,400 feet.

Now, since bodies ascending lose equal velocities in equal times, a body projected upwards will lose all its motion in the same time required to fall back to the earth. Consequently, the shell which was 20 seconds in falling must have been 40 seconds in the air; its *time* of *flight* being 40 seconds.

3. A meteor is seen to explode in the air, and 50 seconds after the fragments reached the earth; required its height at the time of the explosion. *Ans.* 7.57575 miles.

A body in falling 1 second acquired a velocity of 32.2 feet, or, if we allow for the resistance of the atmosphere, a velocity of 32 feet; and the velocity gained after the first second is directly as the time. Therefore,

To find the velocity acquired by a falling body:—

***Multiply the time in seconds by* 32.**

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4. What is the velocity, per second, acquired by a heavy body in falling 10 seconds? *Ans.* 320 feet.

5. What is the velocity acquired by a body in falling 25 seconds? *Ans.* 800 feet.

To find *the time* of *descent*:—

Divide twice the height by the acquired velocity: Or, *Divide the height by* 16, *and extract the square root of the quotient*: Or, *Divide the acquired velocity by* 32.

6. In how many seconds would a body fall 27,000 feet? *Ans.* 41.07 seconds.

7. In what time would a falling body acquire a velocity of 900 feet? *Ans.* 28.12 seconds.

To find the *height*:—

Divide the square of the acquired velocity by 64: Or, *Multiply the acquired velocity by half the time.*

8. Required the height through which a body must fall to acquire a velocity of 1500 feet per second. *Ans.* 35,156 feet.

9. A shell is projected from a mortar in a perpendicular direction with a velocity of 1,400 feet per second; required its greatest altitude, and its time of flight. *Ans.* 30,625 feet, and 87.5 seconds.

To solve the above examples by the *sliding rule*:—

Place 16 *on C over* 1 *on D, and over the time, in seconds, found on D, will be found the height in feet on C; and under the height in feet found on C will be found the time, in seconds, on D.* Or, (because a heavy body will fall one mile in 18.17 seconds,) *Place* 1 *on C over* 18.17 *on D, and over any number of seconds found on D, will be found the height in miles on C; or under any number of miles found on C, will be found the time in seconds, on D.*

10. In how many seconds will a heavy body fall $4\frac{1}{2}$ miles? *Ans.* $38\frac{1}{2}$, nearly.

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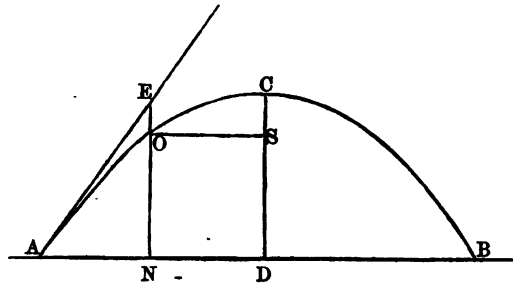
¶ 73. PROJECTILES.

The *range* of a projectile is the distance from the point where it is discharged to that where it strikes the earth, measured on the earth's surface, or the horizontal distance between said points.

All projectiles moving in a vacuum describe a true parabolic curve; and dense bodies, as iron and lead balls, which do not move more than 1,000 feet per second, at the instant of discharge, describe a curve through the air which approaches very nearly to the parabolic curve.



AB represents the *range*, AE the *elevation*, (BAE being the angle of elevation,) and CD the *height*, sometimes called the *greatest height*, or *greatest altitude*, and likewise the *height of projection*. The curve ACB represents the line described by the body projected. ON represents the height of the projectile corresponding to a given horizontal distance, as AN; and EO is the variation of the projectile from AE, the *tangent*, or *line of projection*. [One of the most perfect instances of the parabolic curve is witnessed in columns of water flowing from vessels or from jets, &c., and the rules given in this paragraph, if applied to water-jets, will give very accurate results.] In consequence of the resistance of the air, the path of a body, as has been stated, deviates somewhat from a true parabola into a curve called the *ballistic curve*.

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The greatest *range* of any projectile on a plane (its impetus or velocity of projection being given) is when it is discharged at an angle of 45 degrees; in which case the *height* to which it will rise, is just *half* that to which it would attain, if it were discharged with the same velocity in a *vertical* direction; and its *range* is just *twice the distance* to which it would ascend, if it were projected with the same velocity in a vertical direction; and the *time of its flight* is as much less than it would be, were it discharged in a vertical direction, as the square root of .5, viz. 0.7071065, is less than 1. Therefore,

To find the *range* of a projectile, its greatest *altitude*, and its *time* of *flight*, when discharged with a given velocity at an angle of 45 degrees:—

Divide the square of the velocity of projection by 32, *and the quotient will be the* RANGE. *Divide the* RANGE *by* 4, *and the quotient will be the greatest* ALTITUDE:—*Divide the square root of the* RANGE *by* 4, *and the quotient will be the* TIME *of* FLIGHT:—Or, (because a *range* of 16 feet is passed over in 1 second of time,) *Place* 16 *on C over* 1 *on D, and under the* RANGE *found on C, will be found the* TIME *of* FLIGHT *on D; or over the* TIME OF FLIGHT *found on D, will be found the* RANGE *on C.*

EXAMPLES.

1. A shell was discharged with a velocity of 1,000 feet per second, the angle of elevation being 45 degrees; required the *height* to which it would ascend if projected with the same velocity in a vertical direction, its greatest *altitude*, its *range*, and its *time* of *flight*.

Answers in order,—15,625 feet; 7,812.5 feet; 31,250 feet; and 44.26 seconds.

2. A ball was discharged from a cannon with a velocity of 1,500 feet per second, the gun being elevated at an angle of 45 degrees; required the *range*, the greatest *altitude*, and the *time* of *flight*.

Ans. in order,—70,312.5 feet, or 13.314 miles; altitude, 3.328 miles; time, 66.29 seconds.

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3. The greatest altitude of a shell was 1,600 feet, (the angle of elevation being 45 degrees;) required its *range* and *time* of *flight*. *Ans.* Range 6,400 feet; time 20 seconds.

4. The range of a shot discharged at an angle of 45 degrees, was 12,000 feet; required its greatest altitude, and its velocity of projection. *Ans.* Range 3,000 feet; velocity 617.67 ft.

The *range* of a projectile, discharged with the same velocity, the angle of elevation being the same number of degrees either above or below 45, is the *same;* and when discharged at an elevation of 30 degrees above or 30 below 45, the range will be only half as great as the range at 45 degrees.

5. What will be the range of a shell discharged from a mortar with a velocity of 579 feet per second, the angle of elevation being either 15 or 75 degrees? *Ans.* 5,238 feet.

To find the range of a projectile discharged at any given angle of elevation:—

Multiply the range at 45 *degrees elevation by the* NATURAL SINE *of twice the angle of elevation.* (The sine of any angle of more than 90 degrees is the same as the sine of its supplement. See ¶ 63, page 181.)

To find the time of flight for any elevation:—

Divide the velocity of projection by 16, *and multiply the quotient by the sine of the angle of elevation.*

To find the greatest altitude:—

Multiply half the range at 45 *degrees by the* SQUARE *of the sine of the angle of elevation.*

The sine of one second is .0000048481, very nearly; and the sine of any number of seconds from 1 to 60 may be found very nearly by multiplying the sine of one second by the number of seconds. The sine of one minute is .000290888, very nearly; and the sine for any number of minutes from 1 to 60 may be found nearly by multiplying the sine of 1 minute by the number of minutes.

The following table shows the sine of each degree from 1 *to* 90 *inclusive:—*

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NATURAL SINES.

Deg.	Sines.	Deg.	Sines.	Deg.	Sines.	Deg.	Sines.	Deg.	Sines.
1	.017452	19	.325568	37	.601815	55	.819152	73	.956305
2	.034899	20	.342020	38	.615661	56	.829038	74	.961262
3	.052336	21	.358368	39	.629320	57	.838671	75	.965926
4	.069756	22	.374607	40	.642788	58	.848048	76	.970296
5	.087156	23	.390731	41	.656059	59	.857167	77	.974370
6	.104528	24	.406737	42	.669131	60	.866025	78	.978148
7	.121869	25	.422618	43	.681998	61	.874620	79	.981627
8	.139173	26	.438371	44	.694658	62	.882948	80	.984808
9	.156434	27	.453990	45	.707107	63	.891007	81	.987688
10	.173648	28	.469472	46	.719340	64	.898794	82	.990268
11	.190809	29	.484810	47	.731354	65	.906308	83	.992546
12	.207912	30	.500000	48	.743145	66	.913545	84	.994522
13	.224951	31	.515038	49	.754710	67	.920505	85	.996195
14	.241922	32	.529919	50	.766044	68	.927184	86	.997564
15	.258819	33	.544639	51	.777146	69	.933580	87	.998637
16	.275637	34	.559193	52	.788011	70	.939693	88	.999391
17	.292372	35	.573576	53	.798636	71	.945519	89	.999848
18	.309017	36	.587785	54	.809017	72	.951057	90	1.000000

The natural sines for degrees and parts of a degree, as that for 10 degrees and 20 minutes, may be found very nearly as follows:—Subtract the natural sine of the given number of degrees from the next greater; and say:—As 60 is to the remainder, so is the number of minutes to the part to be added to the sine of the given number of degrees. Thus, .190809—.173648 = .017161. Then, as 60 : .017161 :: 20 to the part to be added to the sine of 10 degrees, viz. 0.005720; consequently, the sine of 10 degrees and 20 minutes = .179368, nearly.

6. A cannon ball is fired with a velocity of 480 feet per second, at an elevation of 75 degrees, and another is fired with the same velocity at an angle of 15 degrees; required the ranges. *Ans.* 3,600 ft., nearly.

7. If a shell be fired with a velocity of projection sufficient to give it a range of 3,600 feet at an elevation of 45 degrees; required its range and time of flight, when discharged with the same velocity at an angle of 32 degrees.

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The natural sine of twice the angle of elevation, (viz. 64 degrees,) is .898794, which multiplied by 3600, gives the required range 3235.65 feet; and the velocity of projection equals the square root of $3600 \times 32 = 339.4$ feet per second; and said velocity, viz. $339.4 \div 16 = 21.212$, which multiplied by .529919, the natural sine of 32 degrees, gives 11.24 seconds, the time of flight; and half the range at 45 degrees, viz. 1800, multiplied by $(.529919)^2$, the square of the sine of the angle of elevation, gives 672.3 feet, the greatest altitude of the projectile.

The ranges are directly proportional to the charges of powder.

8. If a shell range 1,000 yards at an elevation of 45 degrees, how far will it range at an elevation of $30\frac{1}{4}$ degrees, the charge being double? *Ans.* 1,742 yards.

All ranges at the same elevation are directly proportional to the squares of the velocities.

9. If a shell ranges 4,000 feet, the velocity of projection being 480, how far will it range if the velocity be 600 feet?

Ans. 6,250 feet.

Although the above rules give results nearly correct, when the velocity of projection does not exceed 1,000 feet per second; yet the results are far from the truth for greater velocities. This difference is occasioned by the increased pressure of the air on the anterior surface of the projectile, and the greatly diminished or entire removal of a corresponding pressure on the posterior surface; the consequence of which is a retardation of the velocity of the projectile, which greatly diminishes the range.

¶ 74. MEASUREMENT OF DISTANCES BY THE VELOCITY OF SOUND.

It has been proved by careful experiments, that *sound is propagated through dry air at the freezing temperature* (32 de-

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grees Fahrenheit) *at the rate of* 1,090 *feet per second; and through dry air at the temperature of* 60 *degrees at the rate of* 1,125 *feet per second, the velocity increasing with the temperature very nearly at the rate of* 1.25 *feet for every degree of heat.* And since light moves with the astonishing rapidity of nearly 200,000 miles in a second of time, its transmission through short spaces becomes quite insensible, and may be regarded as instantaneous. It is therefore very plain, that when the time elapsed between a flash of lightning, or of the powder of a gun, and the perception of the sound is known, the distance at which the sound was produced may be easily determined.

EXAMPLES.

1. Find the distance of a thunder-cloud, the time elapsed between the lightning and the thunder being 6 seconds, and the temperature of the air 60 degrees. *Ans.* 6,750 feet.

2. An echo of sound was reflected from a rock in 4 seconds after the sound, the temperature of the air being 60 degrees; required the distance of the rock. *Ans.* 2,250 feet.

3. The flash of a cannon is seen at a distance, and 10 seconds after the report is heard; required the distance, the temperature of the air being 42 degrees. *Ans.* 11,025 feet.

The velocity of sound is slightly affected by the variable density of the atmosphere, by the quantity of vapor which it contains, and by the direction of the wind; but none of these causes, nor all of them combined, materially affect the results arrived at by the above data.

¶ 75. PROPERTIES OF THE ATMOSPHERE AND THE EFFECTS OF WIND.

The atmosphere which we breathe is a permanently elastic fluid surrounding our globe to the height of about 45 miles. It is almost wholly composed of two gases, *oxygen* and *nitrogen*, there being in its composition 4 parts of nitrogen to 1 of

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oxygen, by volume, and 75 parts of nitrogen to 23 of oxygen, by weight. Oxygen gas supports respiration, and its existence is necessary to the maintenance of both animal and vegetable life. This gas is highly inflammable, and when a current of it is directed upon burning charcoal, the heat thus developed is so intense as to consume or evaporate iron, tin, copper, and other metals. Nearly half the solid content of the globe is composed of oxygen; and it enters largely into the composition of a great number of bodies. Nitrogen gas is a non-supporter of combustion; and an animal immersed in this gas is killed very shortly by suffocation. The mean pressure of the atmosphere on a square inch, at the surface of the ocean, is 14.6 pounds avoirdupois; consequently, the pressure on a square foot would be 2102.4 pounds, or 2102½ pounds, nearly; and on a circular inch 11.467 pounds, or 11¼ pounds, nearly, and on a circular foot, 1651.22 pounds. This pressure is sufficient to support a column of mercury whose height is 28.6 inches, and a column of water whose height is 33⅓ feet.

The atmospheric pressure continually changes from various causes, as may be shown by the rise and fall of the mercury in the tube of the barometer, its height varying between 27 and 31.8 inches at the level of the sea. The density of the atmosphere decreases in the duplicate ratio of the altitude; that is to say, if at a certain altitude above the earth's surface the weight be half what it is at the surface, then at twice that altitude the density or weight will be only one-fourth of what it is at the surface.

If at the surface of the ocean the height of the mercury in the barometric tube be 30 inches, at an altitude of 90 feet it will be about 29.9, or one-tenth of an inch less, and at the height of twice 90, or 180 feet, it will stand at the height of 29.8 inches, nearly, or one-fifth of an inch less, and so on. It is necessary, however, to take into the account several other particulars in order to render this method of measuring heights by the fall of mercury in the barometer, accurate; for which information we refer the student to Chambers' Mathematics, Part 1, and to Bonnycastle's Trigonometry.

Wind is air in motion. The velocity of wind varies from 0

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to more than 100 miles per hour. A gentle breeze moves with a velocity of 6.8 miles per hour, or 10 feet per second, and exerts a force equal to 0.229 of a pound on a square foot. A very high wind moves 47.7 miles per hour, or 70 feet per second, and exerts a force of 11.207 pounds on a square foot. The wind in a tempest moves 54.3 miles per hour, or 80 feet per second, and exerts a force of 14.638 pounds on a square foot. A hurricane moves 81.8 miles an hour, or 120 feet per second, and exerts a force of 32.926 pounds on a square foot. A violent hurricane moves 102.3 miles per hour, or 150 feet per second, with a force equal to 51.426 pounds on a square foot.

To find the force of wind, its velocity being given:—

Divide the velocity per second by 10, *and multiply the square of the quotient by* 0.229, *and the product will be the force exerted on a square foot in pounds.*

To find the velocity of wind, its force being given:—

Divide the force exerted on one square foot in pounds by 0.229, *and extract the square root of the quotient; multiply this root by* 10, *and the product will be the velocity per second in feet.*

EXAMPLES.

1. What is the force exerted on a square foot of surface by the wind, when its velocity is 36 miles per hour, or 52.8 feet per second? *Ans.* 6.384 lbs.

2. If the velocity of wind be 40 miles per hour, what force will it exert on a square foot? *Ans.* 7.882 lbs.

3. If a horse-power be equal to 400 lbs., to how many horse-power will the wind be equal when exerted on 40 square yards of canvas, its velocity being 60½ miles per hour?
Ans. 16, very nearly.

4. If the force of the wind on the surface of one square foot be equal to 20 pounds; required its velocity.
Ans. 93.453 feet per second.

5. If the pressure of the atmosphere be 14.6 pounds on a

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square inch, what will be its pressure on the area of a circle 20 inches in diameter? *Ans.* 4586.73 pounds.

¶ 75. COMPOSITION AND PROPERTIES OF WATER.

Water is composed of two gases, oxygen and hydrogen; there being 2 parts of hydrogen to 1 of oxygen by volume, and 8 parts of oxygen to 1 of hydrogen by weight.

Hydrogen gas is the lightest of all known ponderable substances, it being 16 times lighter than oxygen, and nearly 14½ times lighter than common air. One cubic foot of hydrogen gas will buoy up, or sustain in the atmosphere, a body weighing 1.16313 oz. avoirdupois; *and* 1,000 *cubic feet would sustain in the atmosphere* 72.6 *pounds.* It is the material used for filling balloons. It burns slowly in the atmosphere; but when burned in connection with oxygen gas, it emits a most intense heat, far exceeding that of a furnace.

Water, which communicates with other water by means of a channel or pipe, *will settle at the same height in all places; and its pressure at any depth is as the depth of the fluid; and it is the same in all directions:* and the amount of pressure on a given surface, its depth being given, will be as the surface; that is, the pressure against the base of any vessel, in all cases, will be the same as that of a column of water in the form of a cylinder of an equal base and height.

If, therefore, the base be 1 square foot, and the height or depth 1 foot, the pressure on the base will be 1,000 oz., or 62½ lbs.; and at twice this depth, whatever be the form of the vessel, the pressure will be twice 1,000 oz., or 125 lbs., and so on. Thus, at the depth of 100 feet the pressure on a square foot would be 6,250 pounds, or 43.403 lbs. on a square inch.

If a vessel be filled with water, and a hole or an orifice be made in the bottom or side, the *water at the same depth will spout out with the same velocity, whether it be upwards, or sidewise, or downwards:* and if it be upwards, it would ascend very nearly to the height of the water above the orifice, were it

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not for the resistance of the atmosphere. In practice, it is found that water under a pressure of 43.403 lbs. to the square inch, will spout upwards about 80 feet. A jet 5 feet high requires the pressure of a column 5 feet 1 inch; and a jet of 100 feet high requires a column of 133⅓ feet.

The resistance which any body meets with in moving through a fluid is as the square of the velocity. For example, if a body moving at the rate of 10 feet per second meets with a resistance equal to 4 pounds, if its velocity be doubled the resistance will be 4 times as great, or 16 lbs.; and if its velocity be trebled, or 30 feet per second, the resistance will be 9 times as great, or 36 lbs.; and if its velocity be increased to 40 feet per second, the resistance will be 16 times 4 pounds, and so on.

Water in freezing expands with a force equal to 35,000 lbs. to the square inch, and enlarges its volume about one-seventeenth.

EXAMPLES.

1. A spherical balloon filled with hydrogen gas is 30 feet in diameter; how many pounds will it sustain in the atmosphere near the earth's surface? *Ans.* 1,016.36 lbs.

2. A cylindric vessel 30 feet deep and 10 feet in diameter is filled with water; required the pressure on its bottom.

Ans. 147,262.5 lbs.

¶ 76. ELASTICITY, OR FORCE OF STEAM.

Water is converted into steam at the temperature of 212 degrees, when it enlarges its volume 1,700 times; that is, a cubic inch of water will occupy 1,700 cubic inches when converted into steam under the pressure of the atmosphere, or a pressure of 14.6 lbs. to the square inch; and its force or power of resisting pressure increases in proportion to the degree of heat communicated to the steam. The following table exhibits the force or elasticity of steam at various temperatures, one atmosphere being equivalent to 14.6 pounds to the square inch, and two atmospheres to 29.2 pounds, and so on.

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Fahrenheit.	Elasticity of steam.	Fahrenheit.	Elasticity of steam.
212°	1 atmos.	380.66°	13 atmos.
234	1½ "	386.94	14 "
250.5	2 "	392.86	15 "
263.8	2½ "	398.48	16 "
275.2	3 "	403.83	17 "
285	3½ "	408.92	18 "
293.7	4 "	413.78	19 "
300.3	4½ "	418.46	20 "
307.5	5 "	422.96	21 "
314.24	5½ "	427.28	22 "
320.36	6 "	431.42	23 "
326.26	6½ "	435.56	24 "
331.7	7 "	439.34	25 "
336.86	7½ "	457.16	30 "
341.78	8 "	472.73	35 "
350.78	9 "	486.59	40 "
358.88	10 "	499.14	45 "
366.85	11 "	510.6	50 "
374	12 "		

See on Steam-Engine, ¶ 79.

¶ 77. PENDULUM.

The pendulum is a heavy body suspended from a fixed point by an inflexible rod. If the pendulum be drawn aside from a vertical position and let fall, it will descend in the arc of a circle, of which the point of suspension is the centre. On reaching the vertical position, it will have acquired a velocity equal to that which it would have acquired by falling vertically through the versed sine of the arc it has described; and if no other than the moving force of gravity acted on the pendulum, it would ascend to a height equal to that from which it fell, and continue to vibrate in the same arc forever. Its passage from the greatest distance on one side to the greatest distance on the other is called the *oscillation*.

CM represents the pendulum rod, and AMB the arc described by the mass or ball of the pendulum. If the ball, M, be drawn aside from its point of rest to A, and let fall, the velocity acquired by the mass at M will be equal to that which

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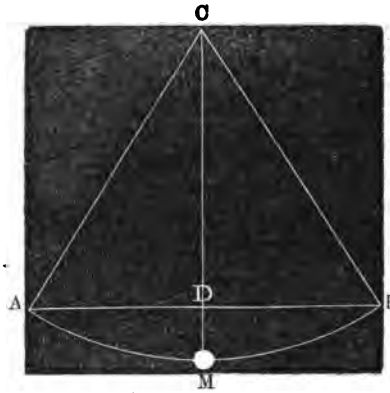

it would acquire in falling through the versed sine of the arc, AMB, or from D to M.

The pendulum performs the double office of measuring time, and of determining the relative force of gravity; *the lengths of pendulums*, vibrating (in different parts of the earth) in the same time, *being directly as the forces of gravity;* and *the forces of gravity*, in different places, *are as their respective distances from the earth's centre*, due allowance being made for the diminution of the force of gravity caused by the earth's rotation on its axis, which at the equator $= \frac{1}{289}$ of the force of gravity.

At the equator a pendulum 39.016 inches in length performs one oscillation in one second; and in latitude 20 degrees north or south of the equator the length of the second's pendulum is 39.0468 inches; in latitude 39 degrees it is 39.1 inches; in New York city (lat. 40° 43′) it is 39.1012 inches; in latitude 44 degrees, 39.11; in latitude 50 degrees, 39.13; in latitude 60 degrees, 39.17; in latitude 80 degrees, 39.22; and at the pole, 39.281 inches.

Let 2 L represent twice the length of a pendulum in feet, and d $16\frac{1}{12}$, the distance described by a falling body in one second, and 1.570796326, the ratio of the semi-circumference of a circle to the diameter; then will the formula $1.570796326 \times \sqrt{\frac{2\,L}{d}}$ express very nearly the *whole* duration of an

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oscillation of the pendulum, whatever its length. Or, when the length of the pendulum is given, to find how many oscillations it shall make in a given time:—

Say—As the pendulum's length in inches is to the length of the second's pendulum for that latitude, (viz. 39.109 inches in New England,) *so is the square of the given time to the square of the required number of vibrations; the square root of which will be the number of oscillations sought.*

EXAMPLE.—Suppose the number of vibrations in a minute is required, the length of the pendulum being 56.34 inches, and the latitude 50 degrees. *Ans.* 50 oscillations.

To find the length of a pendulum that shall perform any number of oscillations in a given time:—

Say—As the square of the given number of vibrations is to the square of the time in seconds, so is the length of the second's pendulum for that latitude to the length of the pendulum sought.

EXAMPLE.—Suppose a pendulum makes 50 vibrations in a minute, in latitude 44 degrees; required its length.

Ans. 56.318 inches.

¶ 78. LAWS OF MOTION.

A body acted on by a single force will describe a straight line; and its velocity will be proportionate to the amount of force applied; that is, if any force applied to a body generates any quantity of motion, double that force will produce double that motion, and treble the force treble the motion, and so on.

The time of describing any space by a body moving with uniform velocity, equals the space divided by the velocity; and the velocity equals the quotient of the space divided by the time.

It is assumed, that every body in motion has the power of communicating motion to another body. The degree of this

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power will depend on the weight of the body and its velocity; consequently, the *power* of any body in motion to overcome any resisting force or obstacle, *is measured by the product of its weight into its velocity*. This product is called its *momentum*. If two bodies move with equal velocities, their momenta will be as their quantities of matter, that is, as their weights; and if their weights are the same, then their momenta will be as their velocities; but if their weights and velocities both differ, their momenta will be in the ratio of the products of their weights multiplied by their velocities. For example, the momentum of a cannon-ball weighing 24 pounds, and moving with a velocity of 40 feet in a second, would be 24 times 40, or 960; and the momentum of another ball weighing 18 pounds, and moving with a velocity of 120 feet per second, would equal 120 times 18, or 2,160; consequently, the effects produced by these balls are in the ratio of 960 to 2,160, or as 4 to 9. From the above data, it is evident that the power requisite to impart to a body a certain velocity must be in proportion to the momentum, that is, to the product of the mass of the body multiplied by the required velocity. Thus, a power equal to 2,000 pounds is required to impart to a mass of 400 pounds the velocity of 5, because 400 times 5 equals 2,000; that is, the power which would impart to a mass of 1 pound a velocity of 2,000, will impart to a body weighing 400 pounds a velocity of only 5. If the moving force continues to act upon a body uniformly after it is set in motion, the velocity of the body will continue to increase, as illustrated in the case of falling bodies. See ¶ 72.

Compound Motion is that produced by the action of two forces on a body at the same time. If two equal forces act upon a body in opposite directions at the same time, it will remain at rest; but if they act upon it at right angles to each other, the body will describe the diagonal of a square, whose sides will represent the spaces through which the body would have passed in the same time, following the impulse of each of these forces. If the two forces which act upon a body at right angles to each other be unequal, the body will describe the diagonal of a rectangle, whose sides will represent the rela-

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tive power of the two forces; and if the directions of the two forces be not at right angles, then the body will describe the diagonal of a rhombus, or the diagonal of a rhomboid, according as the forces are equal or unequal; and the sides of the figure will be proportional to the spaces through which the body would have passed in the same time, following the impulse of each of the forces. For example, if two balls of the same weight, and moving with the same velocity, one moving north and the other east, strike a third ball at the same instant, it will move directly northeast; and if its weight be the same as that of each of the other two, its velocity will be equal to the square root of twice the velocity of one of the balls which generated its motion; but if one of the balls which impinged against the third possessed twice the momentum of the other, then the third would describe the diagonal of a parallelogram or rectangle, whose length would be equal to twice its breadth.

CURVILINEAR MOTION is that produced by the continued action of two separate forces on a body in motion. This kind of motion is illustrated by the motions of the planets, which describe ellipses round the sun. The sun's attraction is called the *centripetal force*, and the tendency which a planet has to move forward in a straight line, is called the *centrifugal* or *tangential force;* and both forces together are called CENTRAL FORCES. The centrifugal force is measured by the product of the weight of the body into its velocity, and the centripetal by the force of attraction towards the point around which the body revolves; the former will therefore vary as the velocity, and the latter will increase or diminish inversely as the square of the distance from the centre of attraction: that is, if the revolving body be 10 times nearer the centre of attraction, the force of attraction will be 100 times greater; or if it be removed 5 times further from the centre of attraction, the centripetal force will be 25 times less. This law applies to both *magnetic* and *electrical* attraction.

The motion of a body on an inclined plane is analogous to that of a falling body, the law being the same, and the only difference being in the degree of velocity, which is as much less as the length of the plane is greater than its perpendicular

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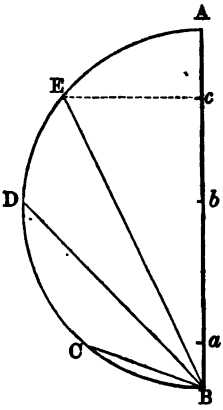
height. If, for example, a heavy body rolls down an inclined plane, *the velocity it acquires in any given time, is to the velocity acquired by a body falling perpendicularly in the same time, as the height of the plane is to its length; and the space described by the body on the plane, is to the space described in the same time by a falling body, as the height of the plane to its length; and the time of a body's descending through the plane, is to the time of falling through the height of the plane, as the length of the plane is to its height; and the velocity acquired by a body in rolling down a plane, is the same as that which it would acquire in falling perpendicularly through its height.* A *curved surface* may be regarded as composed of an infinite number of inclined planes; and consequently, a body in descending along any curved surface will acquire the same velocity as it would in falling perpendicularly through the same height. A body will descend through any chord of a circle in the same time that it would fall perpendicularly through the whole diameter. Thus the times of descending through all the chords of a circle, as EB, DB, CB, and through the diameter AB, are equal; and the velocities of the bodies, which descend the several chords, when they arrive at B, will be equal to that which they would have severally acquired in falling through the versed sines of the arcs cut off by the chords. Thus the velocity of the body descending the chord EB, at B, will be the same as that acquired by a body in falling from *c* to B.



In the above statements and proportions it is supposed, of course, that there is no resistance from friction.

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¶ 79. SIMPLE MECHANICAL POWERS.

The *simple machines* are usually reckoned six in number; the *lever*, the *wheel* and *axle*, the *inclined plane*, the *wedge*, the *screw*, and the *pulley*. The lever and the inclined plane are sometimes called *prime movers*. The wheel and axle is only a modification of the lever; and the wedge and screw are only different applications of the inclined plane.

The LEVER is an inflexible rod moveable about a *fulcrum* or prop, and having forces applied to two or more points in it. It may be regarded as the simplest of the mechanical powers, though not the least useful. The force applied to the lever, for the purpose of moving some object, is called the *power*, and the object to be moved is called the *weight*. The parts of the lever between the fulcrum and the power, and the fulcrum and the weight, are called the *arms* of the lever. Thus, (in Fig. 1,) FW and FP are the *arms* of the lever; W is the *weight*; F is the *fulcrum*; and P is the *power*.

Levers are usually divided into three kinds, according to the relative situations of the power, the weight, and the fulcrum. The first kind has the fulcrum between the power and the weight, (Fig. 1.) The second has the weight between the fulcrum and the power, (Fig. 2.) The third has the power between the fulcrum and the weight. The general principle of the lever is this: When the power and weight are in equilibrium, *the power is to the weight inversely as their distances from the fulcrum*; that is, the power required to balance a given weight may be as many times less than the weight, as the distance between the weight and the fulcrum is less than the distance between the power and the fulcrum. Suppose PF (Figs. 1 and 2) equals 2 feet, and WF equals 1 foot; then a power of 1 pound acting at P will overcome a resistance, or balance a weight of 2 pounds at W; and if PF is 4 feet and FW 1 foot, then 1 pound at P will overcome a resistance of 4 pounds at W; and if PF is 30 feet, and FW 3 feet, then (because PF is 10 times greater than FW) 1 pound at P will bal-

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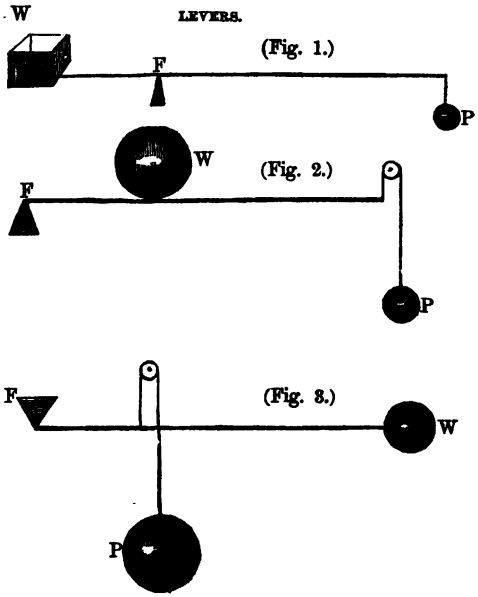
ance 10 pounds at W. When the power and the weight are in equilibrium, the product of the weight into its distance from the fulcrum is equal to the product of the power into its distance from the fulcrum; and if several powers and weights are made to act upon the same lever, they will be in equilibrium, when the sum of the products of the several weights into their distances from the fulcrum, equals the sum of the products



of the several powers into their distances from the fulcrum. In all cases, the weight divided by the power will show the relative length of the arms of the lever. Thus, suppose that it is required to raise, by means of a lever, a weight of 2,000 pounds with a power of 100 pounds; then, since 100 is contained in 2,000 twenty times, the relative length of the arms of the lever must be as 1 to 20; and if the lever used be 10 feet in length, then the shorter arm will be $\frac{1}{21}$ of 10 feet, or 5.7413 inches, and the longer arm will be $\frac{20}{21}$ of 10 feet, or

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114.286 inches. In the application of the lever, (as in all mechanical engines,) the velocity of the power is as much greater than the velocity of the weight, as the weight is greater than the power. Thus, if 1 pound is made to raise 2 pounds, the power must pass through 2 feet to raise the weight 1 foot; and if 1 pound is made to raise 10 pounds, then the power must move through 10 feet to raise the weight 1 foot: therefore, *what is gained in force is said to be lost in velocity;* and this law applies to all machinery, force always being gained at the expense of velocity, and velocity at the expense of power.

The centre of gravity in any body, or between any two bodies, or between any system of bodies, is the point about which the body, or the system of bodies, may be balanced.

EXAMPLES.

1. Suppose a man and a boy are carrying a weight of 400 pounds between them, attached to a lever 7 feet in length; what will be the distance of the weight from each end of the lever, if the man sustains a weight of 300 pounds, and the boy a weight of 100 pounds? *Ans.* 1.75 and 5.25 feet.

2. If on one end of a lever 15 feet in length, a power of 125 pounds balance a weight of 3,000 pounds at the other end of the lever; required the length of the arms.

Ans. $\frac{3}{5}$ and $14\frac{2}{5}$ feet.

3. If the earth be 8,000 miles in diameter, and the moon 2,000 miles in diameter, and both are equally dense bodies, and their distance from each other is 240,000 miles; how far from the centre of the earth will be their common centre of gravity? *Ans.* 3692.3 miles.

4. If the earth and the moon were connected by a lever of the first kind, and the fulcrum were so placed that the moon would just raise the earth; through how many miles must the moon descend in order to raise the earth 1 mile?

Ans. 64 miles.

5. What weight, hung 70 inches distance from the fulcrum of a steelyard, will equipose 900 pounds suspended at 2 inches distance on the other side? *Ans.* 25.714 lbs.

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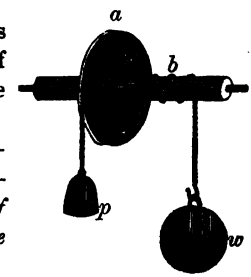
6. A lever is 9 feet long, and one end is fixed by a prop or joint, and a weight of 187½ pounds is hung at the other end; how far from the prop must a power of 281¼ pounds be applied in order to raise the weight? *Ans.* 6 feet.

7. The handle of a pump is 84 inches in length, the bucket is fixed 3 inches from the fulcrum, and the power applied at the other end, 54 inches from the fulcrum, passes through a space of 19 inches; required the *stroke* (or space passed through) by the bucket. *Ans.* 10.57 inches.

8. The arms of the lever of a pump are 54 and 30 inches, the working barrel is 24 feet in height, and its diameter 6 inches; what force must be applied at the end of the longer arm in order to raise the column of water in the barrel after it is full, supposing the column of water to be 24 feet above the surface of the water in the well? *Ans.* 164 lbs., nearly.

The WHEEL AND AXLE *is a perpetual lever*, so contrived as to have a continued motion about a fulcrum. It consists of a wheel having a cylindrical axis passing through the centre, resting on pivots at the extremities, or supported in gudgeons, and capable of revolving. The power is applied to the circumference of the wheel, and the weight or force to be overcome, to the circumference of the axle; and equilibrium takes place, *when the power and the weight are to each other inversely as the radii of the circles to which they are attached.*

In the annexed figure, *a* is the wheel, *b* is the axle, *w* the weight, and *p* the power.



The radius of the wheel corresponds to the longer arm, and the radius of the axle to the shorter arm of the lever.

Therefore, To find the power required to balance a given weight:—*Multiply the weight by the radius of the axle, and divide the product by the radius of the wheel.*

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The windlass, the capstan of a ship, the crank, and any combination of wheels with cogs or teeth, are only modifications of the wheel and axle. *In any combination of wheels with teeth,* the power and weight will be in equilibrium, *when the product of the diameters of all the axles is to the product of the diameters of all the wheels, as the power is to the weight; or, when the product of the number of teeth in these axles is to the product of the number of teeth in the wheels that drive them, as the power is to the weight. And the velocity of the weight is to the velocity of the power, as the product of the diameters of the axles is to the product of the diameters of the wheels; or as the product of the number of teeth in the axles, to the product of the number of teeth in the wheels.*

EXAMPLES.

1. If the radius of a wheel be 6 feet, and the radius of its axle 6 inches, what power will be required to raise a weight of 1,000 pounds? *Ans.* 83⅓ lbs.

2. If in a combination of 4 wheels, the number of teeth in each wheel is 30, and the number of teeth in each of the 4 axles is 5, what power will be required to balance a weight of 15,000 pounds? *Ans.* 11.575 lbs.

3. If in a combination of 5 wheels, the diameter of each wheel is 30 inches, and the diameter of each of the axles 6 inches, what will be the relative velocities of the power and weight? *Ans.* As 3125 to 1.

A PULLEY consists of a small wheel moveable about an axis, and having a groove in its circumference over which a rope or cord passes. The axle of the pulley is supported by a *block,* (or box in which it turns,) and this may be either fixed or moveable. A *fixed* pulley serves merely to change the direction of motion; but several of them may be combined in various ways, by which a mechanical advantage or *purchase* is gained, greater or less, according to their number, and the mode of combination. The purchase gained by any combination may be readily computed by comparing the velocity of the

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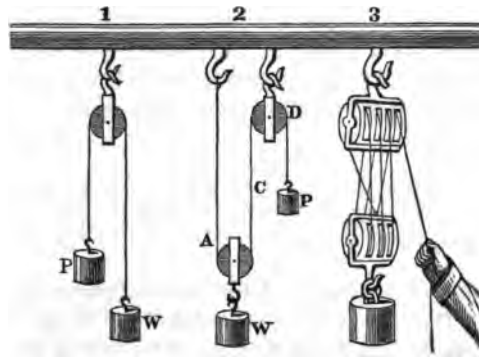
weight with that of the moving power; for in all machinery, *the power is to the weight as the velocity of the weight is to the velocity of the power*. If, therefore, the velocity of the power be to that of the weight as 5 to 1, then a power of one pound will balance a weight of five pounds, and a power of five pounds would balance a weight of twenty-five pounds.

PULLEYS.



The single fixed pulley (represented in Fig. 1) may be considered as a lever, the arms of which are equal to the radii of the wheel, the axis of which is the fulcrum. It is evident, therefore, that in the single fixed pulley the power must be equal to the weight.

Figure 2 represents a fixed pulley D, and a moveable pulley A. In this combination the weight is equally divided between the ropes A and C; and since half of the weight is sustained by each rope, the power will be to the weight as 1 to 2; for, whilst P, the power, exerts a force of 1 pound on the rope C, the hook above the letter A will sustain another pound. In the usual practical combinations of pulleys, *the power is to the weight as* 1 *to twice the number of moveable pulleys;* or, (as in Fig. 3,) *the power is to the weight as* 1 *to twice the number of moveable pulleys, plus* 1; that is, as 1 to the number of ropes connected with the block which contains the moveable pulleys, or as 1 to 7, in the combination represented in Fig. 3.

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In the application of the pulley one-third must be allowed for friction, and sometimes more.

On the INCLINED PLANE, *the power is to the weight as the height of the plane is to its length.* (See ¶ 78.) For example, if the plane be 10 feet in length, and its height 1 foot, then a power of 1 pound will balance a weight of 10 pounds on the plane, and a power of 2 pounds would balance a weight of 20 pounds.

EXAMPLE.—If a barrel of cider weigh 200 pounds, what degree of power will be required to roll it into a wagon 4 feet in height on a plane, or plank, 10 feet in length?

Ans. 80 lbs.

The WEDGE is an instrument sometimes used for raising heavy bodies, but more frequently for dividing or splitting them.

The wedge is composed of two inclined planes placed base to base. Therefore, in the application of the wedge, *the power is to the weight, as half the thickness of the back to the length of either of the sides;* or, as the thickness of the back to the sum of the sides. However, on account of the great friction, the power required to drive the wedge must be to the *resistance* against both sides, as twice the thickness of the back to the sum of the sides. When the wedge is driven by percussion, as is generally the case, its power cannot be estimated with any considerable degree of accuracy. All edge-tools operate on the principle of the wedge, as axes, knives, chisels, &c.

The SCREW consists of a spiral ridge or groove, winding round a cylinder, so as to cut every line on the surface parallel to the axis at the same angle. The screw (when applied to mechanical purposes, or as an engine to raise or press any body) passes through a plate or beam, called the *female* screw, which is so formed that the ridges of the screw fit exactly the hollows of the *interior* or *female* screw. Thus, B represents the screw, C the plate in which the female screw is cut, and

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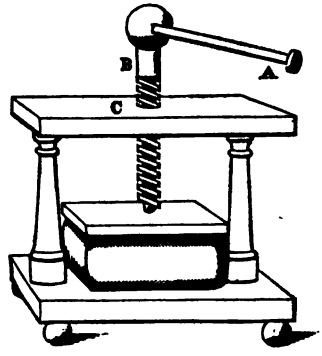
AB the lever by which the screw is turned, the power being applied at A.

THE SCREW.



The principle on which the screw acts is the same as that of the inclined plane. The distance between the contiguous threads of the screw represents the height of the plane, and the circumference of the cylinder, on which the screw is cut, represents its length; consequently, the power is to the weight as the distance between the contiguous threads to the circumference of the cylinder of the screw. The cylinder of the screw is usually turned by a handle or lever; and hence the screw may be regarded as a machine compounded of the lever and inclined plane. Hence, in the practical application of the screw:—*The power is to the weight,* (resistance or pressure,) *as the interval between the adjacent threads to the circumference of the circle described by the power,* acting at the end of the lever.

In the practical application of the screw, it is necessary to allow one-half for friction; that is, the power, in equilibrium with the weight, must be doubled, at least, to produce motion.

The *endless screw* consists of a screw combined with a wheel and axle, in such a manner that the threads of the screw act upon the teeth fixed on the periphery of the wheel. Suppose the power to be applied to the handle of the screw,

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and the weight to be suspended from the axle of the wheel; then, the power is to the weight, as the distance between the threads multiplied into the radius of the axle, is to the circumference of the circle described by the power multiplied by the radius of the wheel.

In all *compound engines*, the power is to the weight in the compound ratio of the power to the weight, in all the simple engines of which it is composed.

EXAMPLES.

1. If the interval between the threads of a screw be 1 inch, and a power of 20 pounds be applied to the end of a lever 30 inches in length; required the weight which the given power will balance. *Ans.* 3,762.92 lbs.

But this power must be doubled to put the weight in motion.

2. If the intervals between the threads of a screw be $2\frac{1}{2}$ inches, and a power of 120 pounds be applied at the end of a lever 5 feet in length; required the weight that will be in equilibrium with the power. *Ans.* 18,095.9 lbs.

3. If the interval between the threads of an *endless* screw be 2 inches, and a power of 100 pounds be applied at the end of a lever 6 feet in length, and the radius of the wheel be 4 feet, and the radius of the axle to which the weight is attached, is 6 inches; required the weight which the given power will balance. *Ans.* 90,478.08 lbs.

4. If the interval between the threads of an endless screw be $2\frac{1}{2}$ inches, and the power be applied at the end of a lever 10 feet long, and the circumference of the wheel be 20 feet, and that of the axle 1 foot, and the weight to be raised is supported on an inclined plane, whose length is 12 feet and height 4; required the weight which will be in equilibrium with a power of 10 pounds. *Ans.* 180,956.16 lbs.

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MACHINERY.

WHEELS, DIAMETERS, AND PITCH OF TEETH.

1. To find the number of cogs, or teeth, in a wheel, the *pitch* of the tooth and the diameter of the wheel being given:—

Multiply the diameter of the wheel by 3.1416, *and divide the product by the pitch of the tooth.* Or, by the sliding rule:—*Set the pitch of the tooth on B to* 3.1416 *on A, and over the diameter found on B, will be found the number of teeth on A.*

2. To find the diameter of a wheel, the *pitch* of the tooth and the number of teeth being given:—

Multiply the number of teeth by the pitch of the tooth, and divide the product by 3.1416. Or, by the sliding rule:—*Set the pitch of the tooth, found on C, under* 3.1416 *on A, and under the number of teeth found on A, will be found the diameter on B; that is,* THE DIAMETER AT THE PITCH LINE.

3. The diameter at the pitch line and the number of teeth being given, to find the pitch of the tooth:—

Multiply the diameter by 3.1416, *and divide the product by the number of teeth.* Or, by the sliding rule:—*Set the diameter found on B, under the number of teeth found on A, and under* 3.1416 *on A, will be found the pitch of the tooth on B.*

EXAMPLES.

1. A wheel is required to be 40 inches in diameter at the pitch line, and the pitch of the tooth, or cog, is 2 inches; required the number of teeth, or cogs, in the wheel.

Ans. 63; or 62.832 teeth.

2. The pitch of a tooth in a wheel must be 2 inches, and the number of teeth 21; required the diameter at the pitch line. *Ans.* 13.4 inches, nearly.

3. A [redacted]0 inches in diameter is to have 146 teeth; what will be the pitch of the tooth? *Ans.* 1.506 inches.

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REVOLUTIONS OF WHEELS.

1. The revolutions of two wheels being given, and the diameter of one of them, to find the diameter of the other:—

Multiply the diameter of the wheel by the number of revolutions it performs in a minute, or in any given time, and divide the product by the number of revolutions which the other wheel is required to perform in the same time, and the quotient will be the diameter. Or, by the sliding rule:—*Set the number of revolutions (performed by the wheel whose diameter is given) under the number of revolutions which the other wheel is required to perform in the same time; then find the given diameter on A, and under it will be found the required diameter on B.*

2. The distance between two shafts, and the number of their revolutions being given, to find the diameters of two wheels that will turn them at the given velocities:—

Multiply double the distance between the two shafts by the number of revolutions which each makes, and the products divided by the sum of their revolutions will give the required diameters. Or, by the sliding rule:—*Set the sum of the revolutions of the two shafts, found on B, under double their distance on A, and over the numbers (expressing the number of revolutions) found on B, will be found the required diameters.*

3. The diameter of a pulley or drum being given, to find the diameter of another pulley or drum that shall make a given number of revolutions, either less or more, in a given time:—

Multiply the given diameter of the pulley or drum by the given number of revolutions which it makes in a given time, and divide the product by the number of revolutions made by the other drum in the same time, and the quotient will be the required diameter.

EXAMPLES.

1. The fly-shaft of a steam-engine, which makes 22 revolutions in a minute, is to give motion (by a pair of spur-wheels)

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to the tumbling-shaft in a mill, which shaft must make 15.5 revolutions in a minute; the distance between the two shafts is 45.5 inches: the diameters of the two wheels are required.

Ans. 53.5, and 37.6 inches.

2. There are two shafts for the purpose of turning machinery, one makes 50 and the other 40 revolutions in a minute; the diameter of the drum upon the shaft, which goes round 50 times in a minute, is 30 inches; required the diameter of a drum that will drive the machinery at the same speed if it make only 40 revolutions per minute.

Ans. 37.5 inches.

3. If the fly-shaft of a steam-engine makes 60 and the governor 38 revolutions in a minute, and the pulley of the shaft be 19 inches in diameter, what must be the diameter of the governor pulley? *Ans.* 30 inches.

PUMPING ENGINES.

The following tables of *gauge points* are adapted to the engineer's sliding rule, and the method of solution here adopted will not apply to the carpenter's rule.

The two following tables of gauge points are for finding the diameters of steam-engine cylinders, that will work pumps from 3 to 30 inches in diameter, and at any given depth in yards. The first table loads its cylinders with 10 pounds upon every square inch of the area in their pistons; and the second table is calculated so as to load the different cylinders with 7 pounds weight upon every square inch of the area of their pistons.

In making use of the two following tables, observe the following rule:—

As the gauge point on A is to unity on B, so is the length of a column of water in yards on C to the diameter of the steam cylinder on D; or set unity on B to the gauge point on A; then against any length of a column of water in yards on C, is the diameter of a steam cylinder that will work the pump on D.

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TABLE of 10 lbs. the inch.

Diameter of Pump.	Gauge Point.	Diameter of Pump.	Gauge Point.
3	115	17	367
4	204	18	41
5	318	19	46
6	458	20	51
7	625	21	562
8	815	22	616
9	103	23	67
10	127	24	732
11	154	25	795
12	183	26	86
13	2125	27	928
14	25	28	99
15	2875	29	107
16	327	30	115

TABLE of 7 lbs. the inch.

Diameter of Pump.	Gauge Point.	Diameter of Pump.	Gauge Point.
3	165	17	528
4	292	18	591
5	457	19	695
6	66	20	731
7	89	21	81
8	117	22	885
9	148	23	97
10	183	24	406
11	222	25	114
12	264	26	124
13	308	27	134
14	358	28	143
15	412	29	154
16	468	30	165

1. What will be the diameter of a steam cylinder sufficient to work a pump 16 inches diameter and 20 yards deep; the piston to be loaded with 10 pounds upon the inch?

In the first table you will find the gauge point for a 16-inch pump, to be 327.

Set unity upon B under 327 on A, and against 20 yards upon C, is 25.5 or 25½ inches upon D, the diameter of the steam cylinder required. When the slide is thus set to its proper gauge point, for any diameter of a pump, the lines C and D are a table for that same diameter; for against any length, in yards, upon C, you have the diameter of the steam cylinder in inches upon D. For example: under 15 on C are 22.1 inches, the diameter, on D; under 20, 25.5; under 25, 28.5; under 30, 31.3; under 35, 33.8; and so of all the rest above or below 20 yards.

2. What will be the diameter of a cylinder to work a pump, 12 inches diameter, at 70 yards deep, and loaded with 7 lbs. on the inch?

In the second table for a 12-inch pump is 264.

Set 1 on B under 264 on A, and under 70 yards on C, is 43 inches on D, the answer.

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This is likewise a table, as under 15 yards on C, are 19.8 inches, the diameter, on D; under 20, 23; under 25, 25.6; under 30, 28.1; and under 35, 30.4.

To find the size of a steam-pan or boiler, sufficient to supply with steam a cylinder of any given diameter:—

Set 1.3 (the gauge point) *on C over* 1 *on D, and over the diameter of the cylinder or piston, found on D, will be found the number of square feet that must be contained in the surface of the water in the boiler,* (whether it be round or square, or any other shape.)

When the slider is *set*, the C line is a table of areas, and the D line is a table of diameters. Thus, over 15 inches on D are 29.2 square feet on C; over 20, 52; over 25, 81; over 30, 117; over 35, 160; over 40, 207; and under 29.2 on C is 15 inches on D, the diameter of a piston requiring 29.2 square feet in the surface of the water in the boiler, &c.

STEAM-ENGINE.

The power of a steam-engine, other things being the same, is as the square of the diameter of the working piston, the power increasing or diminishing as the area of the end of the piston increases or diminishes.

A piston 10 inches in diameter is usually reckoned to be equal to 30 horse-power. Therefore, To find the horse-power of any engine:—

Divide the square of the diameter of the piston by 100, *and multiply the quotient by* 30. Or, by the sliding rule:—*Place* 30 *on C over* 10 *on D, and over the diameter of the piston, found on D, will be found its horse-power on C.* Or, *under any horse-power found on C, will be found the diameter of the piston on D.* Thus, (having *set* the slider,) over 12 on D will be found 43 horse-power on C; over 20 on D, 120 horse-power; over 30, 270 horse-power; over 40, 480 horse-power; and over 60

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inches, 1,080 horse-power. And under 750 horse-power, found on C, will be found 50 inches on D, the diameter of a piston equivalent to a horse-power of 750; and under 2,000 found on C, will be found 81½ inches; and under 3,000 horse-power will be found 100 inches, and so on.

See ¶ 76, and ¶ 81, problems 122 and 123.

FRICTION.

Friction, in the use of machinery, may be reduced to a *minimum* by polishing the surfaces of the axles, gudgeons, and bearings, and keeping them continually well greased, and wetted with water. If, however, the *unguents* used are olive oil, or pure soft tallow or lard, the application of water will be of no service, except it be to keep the running-gears cool. Wood should run upon a bearing of metal, cast-iron upon bell-metal, and wrought-iron upon cast-iron or bell-metal. Friction may likewise be greatly diminished by the application of friction wheels, which are substituted in place of the bearings.

¶ 80. STRENGTH OF MATERIALS.

It is necessary, both in architecture and in the construction of machinery, that due regard should be paid to the strength of the materials employed; and that every part should be made in due proportion to the *stress*, or the *force*, or *pressure* to be endured.

The *strength* of any beam, whether of wood or metal, *is directly as the square of the depth multiplied by the breadth, and inversely as the length.* Hence the strengths of several pieces of timber, their lengths being the same, are to one another as their breadths multiplied by the squares of their depths: if, therefore, their breadths be the same, their relative strengths will be as the squares of their depths; and if their depth be the same, their strengths will be as their breadths. Conse-

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quently, if two beams are of the same length and breadth, and one be twice as deep as the other, it will be 4 times as strong; and if 3 times as deep, it will be 9 times as strong; and if 4 times as deep and twice as broad, it will be 32 times as strong.

If a beam project from a wall, and a weight be suspended from the end of it, the stress suffered by any part of the beam will be as its distance from the weight; and if the weight be augmented, the stress at any point in the beam is augmented in the same ratio; and the stress at any point is measured by the product of the weight into its distance from that point. If the weight be uniformly dispersed throughout the whole length of a beam, the stress upon any point of the beam will be only half as great as it would if the whole weight were suspended at the end of the beam. If a beam be supported at both ends, the breaking weight must be double that required to break a beam of half the length with one end firmly fixed in a wall: the length of a beam supported at both ends may be four times as great as that of the same beam supported at one end only, and having the weight suspended at its other extremity. If a beam be firmly fixed in a wall at both ends, it will sustain $\frac{3}{4}$ as much weight at its centre as it would if the ends were merely supported. When a beam, instead of being laid horizontally, is inclined, its strength is increased nearly in proportion to the angle of elevation; and consequently, it will bear the greatest stress when set in an upright, or vertical position. *The figure of a beam* projecting from a wall, every point in which is equally capable of sustaining a given weight suspended at the end, *will be that of a direct wedge*, whose upper and under sides are parallel with the horizon; or, it may be of the form of a parabola, whose vertex is at the end of the beam, the upper and under sides being curved, and the lateral sides plane surfaces. The weight which is required to break any beam, whether of wood or metal, is considerably greater than that which is required to bend or deflect it; and the quantity by which a beam is bent from its position of rest, is called the *deflection*. Some kinds of timber, (as, for example, ash, fir, and larch,) though capable of sustaining a great weight, are deflected by a comparatively small force or weight. The

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deflection of a beam, other things being the same, is directly as the cube of the length, and inversely as the cube of depth multiplied by the breadth. The deflection of a beam fixed at one end and loaded at the other, is double that of a beam of twice the length supported at both ends, and loaded in the middle with a double weight; consequently, when the weights are the same, the deflection in the first case is to that in the second as 4 : 1; and when the length and weight are both the same, the deflections (which vary as the cubes of the lengths) will be to each other as 32 : 1. If the ends of a beam be firmly fixed in a wall, its deflection under a given weight will be only $\frac{2}{3}$ as great as it would if it were merely supported at the ends.

The *measure* of the *absolute strength* of any material is the greatest weight that a prism one inch square is capable of supporting, acting in the direction of its length; and the absolute strength of any beam, other things being the same, is as the area of the end of the beam. The *absolute strength* of the various woods varies from 6,000 pounds to 17,000 pounds per square inch, elm being the weakest, and ash the strongest of the woods; and the strength of iron, measured by the same standard, varies from 16,300 to 60,000, cast-iron being much the weakest, and malleable iron the strongest.

The resistance of beams to forces tending to crush them, and acting in the direction of the fibres, is usually much less than the force required to draw them asunder: and some of the weakest of the woods, in respect to *absolute strength*, are among the strongest in resisting a *crushing force*. The *resistance* of short pillars of well-seasoned wood, one inch square, to a *crushing force*, varies from 5,000 pounds to 10,000 pounds; and the *resistance* of beams, which are round or square, (that is, their power of resisting a crushing force,) is as the square of the side or diameter directly, and inversely as the square of the length: consequently, the *strength* of *beams* or *pillars* to resist a crushing force, is *measured by dividing the product of the area of the end into the number of pounds of resistance opposed by a pillar one inch square, (of the given substance,) by the square of the length.* Mr. Tredgold found that short pillars

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of cast-iron one inch square would bear a pressure of 93,000 pounds, and would sustain a pressure of 15,300 pounds without permanent alteration. The proper load for square or round iron pillars is, therefore, about 15,000 pounds to the square inch, when the pillar is not more than one foot in length, and the ends flat; for if the ends are round, the pillar will not sustain more than $\frac{1}{3}$ as great a pressure. A vertical cylindrical pillar of cast-iron 34 feet in length and $7\frac{1}{2}$ inches in diameter, will sustain permanently, without bending, a *crushing* weight of about 160,000 pounds; and a vertical column of fir 10 inches in diameter and 16 feet long, will sustain with safety a crushing weight of about 161,000 pounds.

To find the greatest transverse strength of a rectangular beam of New England fir, when fixed at one end and loaded at the other:—

Multiply the breadth by the square of the depth, both in inches, and this product by 1100, *and divide the last product by the length in inches, and the quotient will be the weight in pounds.*

For a permanent load, only $\frac{2}{3}$ of the result obtained by the rule should be taken for fir and the softer kinds of wood, though most of the heavier kinds of wood, as beech, birch, and oak, will sustain with safety, as a permanent load, as great a pressure as that given by the rule; whilst cast-iron will sustain 7 times, and malleable iron 8 times as great a pressure as a permanent load. The strength of a rectangular beam supported at both ends and loaded in the middle, is 4 times as great as that given by the above rule.

STRENGTH OF ROPES, ETC.

Ropes are stronger in proportion to the fineness of the strands of which they are composed. *Damp* hempen ropes are stronger than when *dry;* and such as are tarred are stronger than untarred, twisted than spun, and unbleached than bleached. Silk ropes are nearly three times as strong as hempen or flaxen ropes of the same size.

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To find the strength of a hempen rope:—

Multiply the square of the circumference in inches by* 200, *and the product will be the strength in pounds.

A cylinder of rolls of paper glued together, and presenting at the end an area of 1 square inch, will support a weight of 30,000 pounds; whilst a similar cylinder composed of small hempen strings glued together lengthwise, will sustain 92,000 pounds, its strength exceeding that of the best iron.

A square beam hewn from a round log or cylinder is only $\frac{65}{100}$ as strong as was the cylinder before it was squared. If it be required to hew a beam from a tree that shall possess the greatest *absolute* strength, its length and breadth must be to the diameter of the tree in the ratio of .82 and .58 to 1.

The *lateral* strength of square timber is to that of the tree whence it is hewn, as 10 to 17, nearly. A piece of the best bar-iron 1 inch square will suspend a weight of 77,370 pounds; but thin iron wires, arranged parallel to each other, and presenting a surface at their extremities of 1 square inch, will carry a weight of 126,340 pounds.

EXAMPLES.

1. What weight will break a spar of New England fir, its breadth being 2 inches, depth 3 inches, and length 5 feet?

Ans. 330 lbs.

2. What weight will break a spar of New England fir 10 feet long and 6 inches square, the weight being uniformly distributed throughout the length? *Ans.* 3,976⅔ lbs.

3. Find the proper load for a bar of malleable iron 10 feet long, 2 inches thick, and 3 inches deep. *Ans.* 1,320 lbs.

4. What weight will a bar of cast-iron 6¾ feet long, 1 inch thick, and 3 inches deep, sustain permanently?

Ans. 770 lbs.

5. A joist of New England fir is 25 feet long, 3 inches thick, and 7 inches deep; what weight uniformly distributed over it, will break it when supported at both ends?

Ans. 4,512 lbs.

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6. A birch plank is 10 feet long, 5 inches deep, and 2.24 inches thick; what weight will it permanently sustain at its centre, when both ends are supported? *Ans.* 2,053⅓ lbs.

To find the greatest possible deflection of ash, beech, birch, elm, New England fir, American oak, and teak, before rupture:—

For ash, Multiply the depth of the beam, in inches, by 400; *for beech, by* 615; *for birch, by* 600; *for elm, by* 500; *for fir, by* 757; *for oak, by* 724; *for teak, by* 818; *and divide the square of the length also in inches, by the product, and the quotient will be the greatest deflection, when the beam is supported at both ends; but if the beam is fixed only at one end, the deflection will be eight times greater.*

EXAMPLES.

1. Find the ultimate deflection of an ash plank 2 inches thick, and 40 feet long. *Ans.* 288 inches.

2. A spar of ash 4 inches deep and 6 feet long is fixed at one end, and is broken by a weight applied at the free end; what was the greatest deflection before rupture?

Ans. 26 inches, nearly.

3. A plank of teak (the least flexible of the various woods) is 180.9 inches in length, and 2 inches in thickness; what will be its greatest deflection before rupture, when supported at both ends? *Ans.* 20 inches.

4. A beam of fir is 50 feet long and 5 inches deep; required its greatest deflection when supported at both ends.

Ans. 7 feet, 11.11 inches.

5. Required the strength of a hempen rope 4 inches in circumference. *Ans.* 3,200 lbs.

6. Required the strength of a silk cord 2 inches in circumference. *Ans.* 2,400 lbs.

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¶ 81. MISCELLANEOUS EXERCISES IN MENSURATION

1. How many square feet are there in a floor 16 feet 9 inches square? *Ans.* 280.5625 square feet.

2. What is the area of a board $12\frac{1}{2}$ feet long and 9 inches broad? *Ans.* $9\frac{3}{8}$ square feet.

3. What is the difference between the superficial contents of a floor 28 feet long and 20 broad, and that of two others, of only half its dimensions? *Ans.* 280 feet.

4. It is required to cut off a piece containing a square yard and a half from a plank 26 inches broad; what must be the length of the piece? *Ans.* 6.23 feet.

5. What is the area of a rhombus, whose length is 6.2 chains and perpendicular breadth 5.45 chains?

Ans. 3.36 acres.

6. How many square yards of painting in a rhomboid, whose length is 37 feet, and breadth 5 feet 3 inches?

Ans. $21\frac{7}{12}$ yards.

7. What is the area of a trapezoid, whose parallel sides are $14\frac{1}{2}$ and $24\frac{3}{4}$ feet, and the perpendicular distance between them 8 feet 3 inches? *Ans.* 161.9 feet.

8. What is the length of a rectangle, whose area is 784 feet and breadth 12? *Ans.* $65\frac{1}{3}$ feet.

9. What is the length of a board that contains 18 square feet, its parallel ends being 15 and 9 inches? *Ans.* 18 feet.

10. What is the area of a triangle, whose base is 36 and its altitude 12 rods? *Ans.* 1.35 acres.

11. What is the area of an equilateral triangle whose side is 40 rods? *Ans.* 4.33 acres, nearly.

12. What is the length of a perpendicular falling from either angle of an equilateral triangle on the opposite side, the side of the triangle being 12 chains? *Ans.* 10.3923 chains.

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13. The area of an equilateral triangle is 1 acre, 2 roods, and 15 rods; required the length of its side.

Ans. 24.2687 rods.

14. The area of an equilateral triangle is 24 acres; required the length of its side. *Ans.* 23.542 chains.

15. The base of an isosceles triangle is 25 chains, and each of the other sides 30 chains; how many acres does it contain?

Ans. 34.087 acres.

16. The sides of a scalene triangle are 16, 18, and 24 chains; how many acres does it contain? *Ans.* 14.4 acres.

17. The distance between the feet of two rafters is 20 feet; and one of the rafters is 14 and the other 18 feet in length; required the height of the ridge above the plates on which the rafters stand. *Ans.* 12.2376 feet.

18. What is the area of a regular pentagon, whose side is 25 feet, and its apothegm 17.2 feet? *Ans.* 1,075 feet.

19. How many square rods are contained in an octagon, whose side is $12\frac{1}{2}$ feet? *Ans.* 2.77 rods.

20. The area of a pentagon is 4 acres; required its perimeter. *Ans.* 96.435 rods.

21. The wheel of a carriage turns once and a half in a rod; required its diameter. *Ans.* 3.5 feet.

22. Required the value of a circular garden at 8 cents per square foot, its diameter being 6 rods. *Ans.* $615.8164.

23. If a horse be tied to a stake by a cord, what must be its length in order that he may feed on an acre of ground?

Ans. 235.504 feet.

24. What is the radius of a circle that contains 2 acres?

Ans. 2.5231 chains.

25. The diameter of a circle is 36 feet 3 inches; required the length of an arc of 30 degrees. *Ans.* 9.49 feet.

26. The chord of an arc of a circle is 12 feet, and its height, or versed sine, 2 feet; required the diameter of the circle.

Ans. 20 feet.

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27. The versed sine of a circular arc is 2 feet, and the diameter of the circle 18 feet; required the length of the chord which cuts off the arc. *Ans.* 11.3136 feet.

28. How many perches of stone will be required to build a wall 2 feet thick and 4½ feet high, around a circular garden which contains half an acre? *Ans.* 288.3 perches.

29. The area of a rectangle is 10 acres, 3 roods, and 8 rods, and its diagonal is 60 rods; required the sides.
(See ¶ 65, problem 13.) *Ans.* 48 and 36 rods.

30. What is the area of a circular segment, its chord being 30, and versed sine 5 rods? *Ans.* 102.187 rods.

31. Required the area of a circular zone, the chord of one of the segments being 8 rods, the versed sine of the other segment 4.8 rods, and the diameter of the circle 12 rods.
Ans. 62.5 rods.

32. Suppose a tree 100 feet in height to be broken off by the wind, and that the top of the tree strikes the ground 40 feet from its base, whilst the other end of the part broken off rests on the top of the stump; required the length of the part broken off. (See ¶ 65, problem 1.) *Ans.* 43⅓ feet.

33. There are two parallel walls standing on a plane, and the foot of a ladder, whose length is 26⅔ feet, may be so planted that it will just reach the top of each wall; one of the walls is 22 and the other 14 feet in height: I require their distance from each other. *Ans.* 37 feet and 9.24 inches.

34. Required the side of the greatest square stick, the side of the greatest equilateral triangular prism, and also the side of the greatest octagonal prism, which can be cut from a round log, whose diameter at the smaller end is 18 inches.
Ans. in order,—12.73; 15.588; and 6.5 inches.

35. Required the side of an octagon cut from a square 2 feet on a side. *Ans.* 10 inches, nearly.

36. Required the diameter of a circle, in which a decagon 4 inches on a side may be inscribed. *Ans.* 12.945 inches.

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37. Required the area of a crescent, whose chord is 24, and the versed sines of the arcs 2 and 10. *Ans.* 148.2374.

38. What is the area of an ellipse, whose major and minor axes are 60 and 40 rods? *Ans.* 11.781 acres.

39. What is the circumference of an ellipse, whose axes are 10 and 100? *Ans.* 204$\frac{1}{3}$, nearly.

40. The area of a circle is 4 acres less than the area of its least circumscribing square; required its diameter.

Ans. 13.6525 chains.

41. A log of wood is 15 inches broad and 11 thick; what length of it will make 10 cubic feet? *Ans.* 8 ft. 8$\frac{8}{11}$ inches.

42. A round cistern is 26.3 inches in diameter; what must be the diameter of another to contain twice as much, its depth being the same? *Ans.* 37.19 inches.

43. What will be the expense of painting a conical church spire, at 8*d*. per yard, the circumference of the base being 64 feet, and its slant height 118 feet? *Ans.* £13 19*s*. 8$\frac{4}{9}$*d*.

44. What will be the expense of gilding a sphere, 6 feet in diameter, at 6$\frac{1}{4}$ cents the square inch? *Ans.* $1,017.878.

45. How many three-inch cubes can be cut out of a cubic foot? *Ans.* 64.

46. The numbers expressing the solidity and the surface of a sphere are the same; what is its diameter? *Ans.* 6.

47. How far will a point in the circumference of a wagon-wheel move whilst the wagon is drawn one mile over a smooth level road, the diameter of the wheel being 4 feet?

Ans. 6,722.688 feet.

48. If two wheels, whose diameters are 4 and 5 feet, be placed on the ends of an axletree 20 feet long, and set rolling on a plane, what will be the diameter of the circle described by the larger wheel? *Ans.* 200 feet.

49. How far may the lamp of a lighthouse, 150 feet high, be seen at sea, when the eye of the observer is elevated 100 feet above the surface of the water? *Ans.* 30 miles, nearly.

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50. What is the side of an equilateral triangle, whose area cost as much for paving, at 8*d.* per foot, as the fencing of the three sides at a guinea a yard, the value of a guinea being 21 shillings? *Ans.* 72.746 feet.

51. There are two round logs of the same length, and their diameters are 2 and 5 feet; how much more timber is contained in the larger than in the smaller log?

Ans. 6¼ times as much.

52. Required the side of a cubic box, whose capacity is 10 bushels. *Ans.* 27.808 inches.

53. What is the length of a diagonal between the opposite corners of a cube whose side is 20 inches?

Ans. 34.641 inches.

54. How many bushels will a bin hold, its length being 5 feet 6 inches, its breadth 4 feet 9 inches, and its depth 3 feet 9 inches? *Ans.* 78.724 bushels.

55. The side of an equilateral triangular prism is 18 inches, and its solidity 39 feet; required its length.

Ans. 40.029 feet.

56. A round cistern is 5 feet 6 inches in diameter; what must be its depth to hold 20 wine hogsheads?

Ans. 7 feet 2 inches, nearly.

57. How many bricks will it take to build a wall 10 feet high, and 250 feet long, if the bricks be 10 inches long, and 4 courses make one foot in height, the breadth of the wall being equal to the breadth of 3 bricks? *Ans.* 36,000.

58. The same number expresses the solidity and convex surface of a cylinder; what is its diameter? *Ans.* 4.

59. A gentleman has a garden 100 feet long and 80 feet broad; required the breadth of a walk extending round two sides, which shall cover half the area. *Ans.* 25.968 feet.

60. What must be the diameter of a bushel measure, its depth being 7½ inches? *Ans.* 19.1067 inches.

61. The ditch around a fortification is 1,000 feet long, 9

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feet deep, and 20 feet broad at the bottom and 22 at the top; how many ale gallons of water will fill the ditch?

Ans. 1,158,128.

62. The solidity of a cone is 20 feet, and the diameter of its base 20 inches; required its length. *Ans.* 27½ feet.

63. The altitude of a cone is 10 feet, and its solidity is equal to 21 bushels; required the diameter of its base.

Ans. 37.9137 inches.

64. How many *wine*, and how many *ale* hogsheads will a cistern hold, whose diameters at the top and bottom are 4¾ and 5 feet, and its depth 8 feet?

Ans. 17.734 wine, and 16.9 ale hhds.

65. What is the capacity of a box whose dimensions are, at the top 7 and 6 feet, at the bottom 5 and 3 feet, and its depth 4 feet, its shape being that of a prismoid, or frustum of an indirect wedge? *Ans.* 110 feet.

66. Find the solidity of a prismoid, the length and breadth of its greater end being 24 and 16 inches, those of the less 16 and 12 inches, and its length 10 feet. *Ans.* 19.624 feet.

67. Required the contents of a sphere in feet and bushels, its diameter being 2 feet 6 inches.

Ans. 8.179 feet; 6.58 bushels.

68. The expense of gilding a ball at $1.80 per square foot is 40 dollars; required its diameter. *Ans.* 2.6596 feet.

69. If a cone 40 inches high is cut into three equal parts by planes parallel to the base; what must be their lengths?

Ans. 27.734; 7.21; and 5.056 inches.

70. The frustum of a square pyramid is 18 feet long, and the sides of its ends are 1 and 3 feet, and it is divided into three equal portions; what must be the length of each?

Ans. 3.269; 4.559; and 10.172 feet.

71. The solidity of the frustum of an equilateral triangular pyramid is 4 feet, and the sides of its ends 14 and 8 inches; required its length. *Ans.* 10.727 feet.

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72. Required the diameter of a spherical vessel whose capacity is 1 bushel. *Ans.* 16.04 inches.

73. What is the side of the greatest cube that can be cut from a sphere 18 inches in diameter? *Ans.* 10.392 inches.

74. What must be the diameter of a cylindric vessel 3 feet deep, that will contain twice as much as another 28 inches deep and 46 inches in diameter? *Ans.* 57.37 inches.

75. A cubic foot of brass is to be drawn into a cylindric wire $\frac{1}{40}$ of an inch in diameter; what will be the length of the wire? *Ans.* 97,784.797 yards.

76. A bowling-green 300 feet long and 200 broad, is to be raised one foot higher by means of earth dug from a ditch to be made round it; what must be the depth of the ditch, its breadth being 8 feet? *Ans.* $7\frac{3}{8}\frac{3}{6}$ feet.

77. A circle 60 inches in diameter is to be divided into three equal portions by means of two concentric circles; what must be their diameters? *Ans.* 34.641; and 48.988 inches.

78. The diameter of a circle is 20 rods; what is the area of the inscribed square? *Ans.* 200 rods.

79. The diameter of a sphere is 12 inches; it is required to find the diameter of another three times as large.

Ans. 17.307 inches.

80. Required the weight of a bushel of water.

Ans. 77.778 lbs.

81. Required the weight of an oak stick 12 feet long and 31 inches square. *Ans.* 2.325 tons.

82. How many lbs. of butter will a square box hold, its side being 6 inches and its depth 7 inches? *Ans.* 10 lbs. 2 oz.

83. What is the length of a pendulum which vibrates once in 7 seconds, in lat. 44? *Ans.* 159.7 feet, very nearly.

84. What is the ullage of a lying cask, whose capacity is 60 wine gallons, the bung diameter being 27 inches, and the depth of the liquor under the bung 10 inches?

Ans. 20.2776 gallons.

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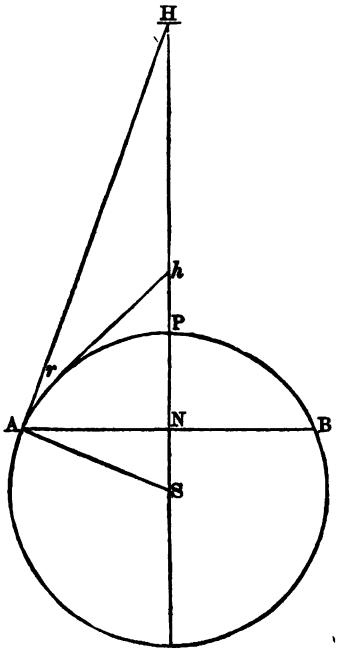
85. What is the distance between the opposite corners of a room 20 feet long, 15 broad, and 10 feet high?

Ans. 27 feet, nearly.

86. What is the diameter of a circle, the versed sine of whose arc is 8 feet, and the chord 40 feet? *Ans.* 58 feet.

87. If the diameter of the earth is 7,912 miles, how high above the earth must a person be elevated at the north pole, in order to see an object on the earth's surface in latitude 45 degrees? and how high in order to see one-third of the earth's surface? *Ans.* 1,593.62 miles; and 7,912 miles.



The first question (problem 87) is easily solved, since the tangent *rh* may be proved to be equal to radius, or 3956 miles, it being half of one side of the circumscribing square. In the second question:—Let APB represent one-third of the earth's surface; then NP will be one-third of the earth's diameter; and hence NS must be one-sixth of the earth's diameter, or 1318$\frac{2}{3}$; therefore, in the right-angled triangle ANS, we have the hypotenuse AS, viz. 3956 miles, and NS= 1318$\frac{2}{3}$ miles; hence we may find AN, which equals 3729.75 miles. Now the triangles ANS and AHN are similar, and their sides proportional. Consequently, NS is to AN, as AN is to HN; that is, 1318$\frac{2}{3}$: 3729.7525 : : 3729.7525 is to HN. Hence,

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HN=10549.33 miles, from which, if we subtract PN, viz. 2637.33 miles, the remainder is HP, viz. 7912 miles.

88. A coppersmith is required to make a flat-bottomed kettle, of the form of a conic frustum, to contain 13.64277 ale gallons, the depth of the kettle to be 12 inches, and the diameters of the top and bottom to be in the ratio of 5 to 3; what are the diameters? *Ans.* 25 and 15 inches.

89. A piece of marble, of the form of a conic frustum, has its end diameters $1\frac{1}{2}$ and 4 feet, and its slant side is 8 feet; what will it cost at 3 dollars the cubic foot?

Ans. $150.5234.

90. The price of a ball at 1 cent the cubic inch is the same as the cost of gilding it at 3 cents the square inch; what is its diameter? *Ans.* 18 inches.

91. A garden, 500 feet long and 400 broad, is surrounded by a terrace walk, the surface of which is one-eighth of that of the garden; what was the breadth of the walk?

Ans. 13.6809 feet.

92. A reservoir is supplied from a pipe 6 inches in diameter; how many pipes 2 inches in diameter would discharge the same quantity, supposing the velocity of the water to be the same? *Ans.* 9 pipes.

93. A pipe 4 inches in diameter is sufficient to supply a town with water; what must be the diameter of a pipe which will supply it when its population is increased by a half, the velocity being the same? *Ans.* 4.899 inches.

94. When the pressure of the atmosphere is 15 pounds on a square inch, what would be its pressure on the surface of a man's body, supposing it to be 8 square feet?

Ans. 17,280 pounds.

95. The sanctuary of Butus in Egypt was formed of one stone, in the form of a cube of 60 feet on a side, open at the top, and hollowed so that it was everywhere 6 feet thick; required its weight, at the rate of $157\frac{1}{2}$ pounds the cubic foot.

Ans. $6,439\frac{1}{2}$ tons.

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96. If the diameter of the earth be 7,912 miles, what will be the length of a degree of longitude in latitude 45 degrees?

Ans. 48.82 miles.

97. If the radius of a circle is 6 miles, what is the length of an arc corresponding to 1 second? *Ans.* $1\frac{7}{8}$ inch, nearly.

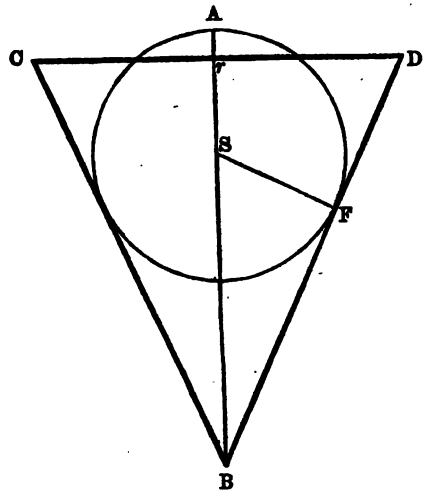
98. The area of a rectangle is 120 acres, and its length exceeds its breadth by 40 rods; required its length and breadth. (See ¶ 65, problem 12.) *Ans.* 40 and 30 chains.

99. Three equal circles touch each other, and leave a piece of ground between their circumferences, whose area is 1 acre; required the area of each of the circles. (See ¶65, problem 18.)

Ans. 19.4772 acres.

100. A conical glass, whose depth is 6 inches, and the diameter of its mouth 5 inches, being filled with water, and a sphere 4 inches in diameter, of greater specific gravity than water, being put into it; how much water will run over?

Ans. 26.27215 inches.



Find DB; then, because the triangles D*r*B and SBF are similar, D*r* is to DB as SF is to SB. Therefore, 2.5 : 6.5 :: 2 :

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SB=5.2; and 6—5.2=.8, or rS; and 2—.8=1.2, or Ar; and the solidity of the sphere, viz. 33.5104—7.23825, the solidity of the segment Ar, leaves the part immersed, viz. 26.27215.

101. If a ball fall into a conical glass 21 inches deep, and the diameter of its mouth 14 inches, and the size of the ball be such that it is barely immersed in the fluid with which the vessel is filled, what will be its diameter; and how much liquor will be driven out of the vessel?

Ans. 10.09063 inches; and 2.3288 wine gallons.

It may be easily proved by comparing the similar triangles, that SF, the radius of the sphere, equals $7\times SB\div 22.135943$; and hence, $21-SB=\frac{7\times SB}{22.135943}$; and $29.135943\times SB=464.8548$; and SB=15.954684; consequently, SF=5.045315 inches.

102. If a cone equal to that in example 100, be one-fifth full of water; what portion of the vertical diameter of a sphere 4 inches in diameter, will be immersed? *Ans.* 0.546 inch.

103. If a cone equal to that described in example 100, be one-fifth full of water; what is the diameter of a sphere which, when placed in it, would just be covered with water?

Ans. 2.446 inches.

First find the diameter of the greatest sphere which could be immersed in the cone, and its solidity; these are $3\frac{1}{5}$, and 19.3926 inches; and the content of the sphere subtracted from 39.27, the content of the cone, leaves 19.8774 cubic inches, the quantity of water required to immerse it. Then, as the quantity of water required to immerse this sphere is to the cube of its diameter, so is the given quantity, viz. 7.854, (one-fifth of the content of the cone,) to the cube of the diameter of the sphere which it will immerse.

104. If the mouth of a conical vessel be 24 inches in diameter, and its depth 33.94112 inches, and it is one-third full of wine; required the diameter of a sphere that will just be immersed in the liquor if it fall into the vessel.

Ans. 14.8455 inches.

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105. A shell is fired from a mortar in a vertical direction with a velocity of 900 feet per second; required its time of flight, and its greatest altitude above the earth.

Ans. 56¼ seconds; 12,656 feet.

106. A ball is fired from a cannon at an elevation of 45 degrees with a velocity of 1,200 feet per second; required its greatest altitude, its range, its time of flight, and the length of the curve which it describes.

Ans. 2.13; and 8.52 miles; 5 minutes; and the length of the curve described through the air = 9.838 miles.

107. What 5 weights will weigh any number of pounds from 1 to 121? *Ans.* 1, 3, 9, 27, and 81.

108. What is the length of the side of a cubical box, into which if we introduce a globe whose diameter is equal to the side of the box, it will require 2150.42 cubic inches to fill the vacancies? *Ans.* 16.5266 inches.

109. The distance of a certain point, in an equilateral triangle, from two of the angles is 20 rods, and from the third angle 31 rods; required the side of the triangle.

Ans. 39.5 rods.

110. A gentleman owns a square piece of land 50 rods on a side, in the centre of which he forms a circular fish-pond, of such a size that it will just meet the arcs of two circles described with a radius of 50 rods each, and having their centres at the opposite corners of the square; required the diameter of the pond. *Ans.* 29.28932 rods.

111. If in a quadrant, whose radius is 200 rods, two semicircles be described with a radius of 100, and having their centres at the centres of the radii of the quadrant, what will be the area of the greatest circle that can be described between their arcs and the arc of the quadrant? *Ans.* 12.8156 acres.

(See ¶ 65, Problem 28.) The diameter may be found by multiplying the radius of the quadrant by the decimal .25547829.

112. The sides of a triangular prism are 10, 15, and 18

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inches; required the side of the greatest square prism that can be cut from it. *Ans.* 5.6946 inches.

(See ¶ 65, problem 19; and ¶ 19, example 13.)

113. The area of a right-angled triangle is 25,600 rods, and the sides are in geometrical proportion; now, if 1.27202066 represent the shortest side, I require the sides of the triangle.

Ans. 200.6263; 255.2008; and 324.6206 rods.

114. The parallel sides of a trapezoid are 16 and 24 rods, and its area 5 acres; required the length of a line parallel to the parallel sides, that will divide it into two equal parts.

Ans. 20.39608 rods.

115. The major axis of the earth's orbit is 191 millions of miles, and its minor axis is 190.976323 miles; required the sun's distance from the centre of the ellipse; that is, the eccentricity of the earth's orbit. *Ans.* 1½ million of miles.

116. The axes of the earth's orbit being the same as stated in the former exercise; what is the parameter of its orbit?

Ans. 190,952,879½ miles.

117. Should a fortunate miner find a sphere of pure gold 4 inches in diameter, in the placers of California, I demand its true value. *Ans.* $6683.268.

118. A slide on the side of a mountain in Switzerland, is three miles in length, and its perpendicular height above the lake in which it terminates is one and a half mile; what will be the momentum of a body weighing 500 pounds at the foot of the slide? *Ans.* 355,977.6 lbs.

119. A bomb requires two pounds of loose powder to fill the internal concavity; it is of cast-iron, and its weight 30 pounds; required the thickness of the shell.

Ans. 0.9853 inch.

120. What weight will a balloon, filled with hydrogen gas, sustain in the atmosphere near the earth, its form being that of a parabolic spindle 50 feet long, and its diameter 20 feet.

Ans. 608.214 lbs.

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121. A cast-iron shell, whose external diameter is 30 inches, will just float in pure water; required its thickness.

Ans. 0.70799 inch.

122. If steam be heated to the temperature of 358.88 degrees, what force will it exert on the end of a piston 10 inches in diameter? *Ans.* 11,466.84 lbs.

123. If a horse-power be equal to 400 pounds, to how many horse-power will a steam-engine be equal, the diameter of the piston being 30 inches, and the temperature of the steam 418.46 degrees? *Ans.* 516.0078.

124. A careless baker baked a hemispherical loaf of bread till it was half crust; the crust was of equal thickness throughout the whole loaf, the diameter of its base was 10 inches, and its height, of course, 5 inches; required the thickness of the crust. *Ans.* 0.66984 inch.

125. The equatorial diameter of the earth is 7,925 miles, and the ratio of the centrifugal force at the equator to the force of equatorial gravity, is as 1 to 289, or $\frac{1}{289}$; I demand the polar diameter of the earth. *Ans.* 7,898.772 miles.

126. Required the diameter of a globe from which an octahedron may be cut, whose side is 12 inches.

Ans. 16.9705 inches.

127. A tinker is required to make a tin can that will hold 9 pounds of powder well shaken; its form is required to be that of a conic frustum, its top and bottom diameters being 5 and 7 inches; what must be its altitude? *Ans.* 8.0733 inches.

128. If the equatorial diameter of the earth be 7,925 miles, what will be its diameter between the 44th degrees of north and south latitude, the ratio of the centrifugal force to the force of gravity in latitude 44° being as 1 to 400?

Ans. 7,913.52 miles.

129. A spile-hammer, whose weight is 250 pounds, falls 16 feet; required its force, or momentum.

Ans. 8,000 pounds.

130. If a charge of 6 pounds of powder is sufficient to im-

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pel a ball over a range of 3,600 feet, what charge will be required to give the ball a range of 4,500 feet?

Ans. 7.5 pounds.

131. The resistance of the atmosphere to a 12-pound ball, moving with a velocity of 25½ feet per second, is half an ounce avoirdupois; and the resistance for velocities less than 1,100 feet, being nearly proportional to the squares of the velocities, it is required to find the resistance opposed to an 18-pound ball moving with a velocity of 1,000 feet per second.

Ans. 62.876 pounds.

132. The breadth of a ditch in front of a tower is 48 feet; and from the outer edge of the ditch, the angle of elevation of the top of the tower is 53° 20′; what is the height of the tower? *Ans.* 64.47 feet.

133. Required the height of a tower, a horizontal base being measured of 245 feet, and the angle of elevation being 35 degrees 24 minutes. *Ans.* 174.11 feet.

134. If three trees be so planted that the angles of the triangle, at the corners of which they stand, are to each other as the numbers 1, 2, and 4, and that a line of 100 yards will just go round them; required their distances from each other.

Ans. 19.8; 35.69; and 44.5 yards, nearly.

One-seventh, two-sevenths, and four-sevenths of 180 degrees, will be the angles, viz. 25° 42′$\frac{5}{6}$ and 51° 25′$\frac{4}{6}$ and 102° 51′$\frac{4}{16}$; and their natural sines are .43388 and .78183 and .97493, the sum of which is 2.19064. Then, as 2.19064 : 100 :: 0.43388 to the side of the triangle opposite the least angle.

135. In order to find the height of a conic pyramid, a base line was measured from its base of 130 feet, and the angles of elevation of the top of the pyramid, measured at the extremities of the base, were 31° and 46°; required its *slant*, and its *perpendicular* height. *Ans.* 258.69, and 186.08 feet.

The angle of elevation at the base of the pyramid, viz. 46°, subtracted from 180° leaves 134°, the angle between the base line and slant height; consequently, 180—(134+31,) or 15 degrees, equals the angle opposite the base. Hence, as

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.258819 : 130 : : 0.515038 to the slant height. And, as 1 : 258.69 : : .719340 to the altitude of the pyramid.

136. Required the height of an inaccessible tower on the opposite side of a river, the length of the horizontal base being 170 feet, and the angles of elevation at its extremities 32° and 58°. *Ans.* 174.27 feet.

UNGULAS.

Ungulas are portions cut from pyramids, prismoids, cylinders, and cones, by plane sections not parallel to the base.—Ungulas cut off from pyramids, or from triangular or rectangular prisms, or from prismoids, are wedges, or the frustums of wedges, and their solidities may be found accordingly. See ¶ 43 and 44.

The content of a *cylindric ungula*, cut off by a plane perpendicular to the base, *may be found by multiplying the area of the end by its length.* The end will of course be the segment of a circle, the area of which may be found by the rules for circular segments. See ¶ 25.

To find the solidity of a CYLINDRIC UNGULA *cut off by a plane inclined to the base:—*

Multiply the area of the base of the ungula by the difference between the diameter of the cylinder and twice the versed sine of the base of the ungula, and subtract this product from one-sixth of the cube of the chord of the base of the segment, if said base be less than a semicircle, but add this product to one-sixth of the cube of said chord when the base is greater than a semicircle; multiply the remainder in the former, or the sum in the latter case, by the length of the ungula, and divide the product by twice the versed sine of the segment's base.

137. Find the solidities of the two wedges into which a frustum of a rectangular pyramid is divided by a plane passing through two of the shorter opposite edges of its ends, the length and breadth of its base being 45 and 30, those of its top 36 and 24, and its height 40.

Ans. 25,200 and 18,720.

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138. The length of a cylindric ungula is 10 feet, the diameter of the cylinder 18 inches, and the section is 6 inches distant from the axis and perpendicular to the base; what is the solidity of the two ungulas? *Ans.* 1.9359; and 17.6715 feet.

139. Find the solidity of a cylindric ungula cut off by a plane inclined to the base, the diameter of the cylinder being 25, the length of the ungula 60, and the versed sine of its base 5. *Ans.* 1709.92.

140. A cylindric vessel 10 inches in diameter, and partly filled with wine, is inclined till the horizontal surface of the fluid leaves 8 inches of the diameter of the bottom dry, and meets the side of the vessel 24 inches from the bottom; required the number of cubic inches of wine in the vessel.

Ans. 109.434 inches.

141. Suppose that the fluid in the same vessel leaves only 2 inches of the bottom diameter dry, and that it rises to the same height as before; what is the quantity of wine?

Ans. 734.2169 inches.

CONIC UNGULAS are *elliptic*, *parabolic*, or *hyperbolic*, according as the plane, which cuts off the ungula, is an ellipse, or portion of an ellipse, or a parabola, or an hyperbola. (See paragraphs 28, 29, and 30.)

To find the solidity of a CONIC UNGULA, *cut off by a plane passing through the opposite edges of the ends of the conic frustum:—*

Extract the square root of the product of the two diameters of the conic frustum; multiply this root by the less diameter, and subtract the product from the square of the greater diameter; divide the remainder by the difference of the diameters, and multiply the quotient by .2618, and multiply this product by the product of the greater diameter into the altitude of the frustum, and the result will be the solidity of the greater ungula, which, subtracted from the solidity of the frustum, will give the content of the less ungula.

142. Find the solidity of the greater ungula of a conic frus-

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tum, cut off by an elliptic plane passing through the opposite ends of the frustum, the diameters of the ends being 15 and 9.6 inches, and the height of the frustum 20 inches.

Ans. 1596.98 inches.

143. A glass vessel is 5.7 inches deep, and its top and bottom diameters are 3.7 and 4.23 inches; supposing it to be filled with wine, and that a quantity of it is poured out till the remainder just covers the bottom; required the quantity left in the vessel, and the quantity poured out.

Ans. 38.773 and 31.712 inches.

To find the solidity of ELLIPTIC UNGULAS *of a conic frustum made by a section cutting off a part of the base:—*

Find the solidity of a cylindric ungula of the same base and altitude, (cut from a cylinder whose diameter is equal to the greater diameter of the conic frustum;) then multiply the square of the difference of the diameters of the frustum by its altitude, and one-tenth of this product subtracted from the content of the cylindric ungula, will give the content of the required ungula, nearly. Or, the solidity may be found by the rule for the hyperbolic ungula. See page 271.

144. Find the solidity of an elliptic ungula, cut from a conic frustum whose top and bottom diameters are 15 and 9.6 inches, its altitude 20 inches, and the versed sine of the base of the ungula 10 inches. *Ans.* 1038.838 cubic inches.

145. A bucket whose top and bottom diameters are 19.2 and 30 inches, contains a quantity of fluid, which, when the vessel is inclined, just reaches the lip, and leaves 10 inches of the bottom diameter dry; how many cubic inches of fluid are there, supposing the depth of the vessel to be 20 inches?

Ans. 4155.35 inches.

146. A pot of the form of a conic frustum is 10 inches deep, and its top and bottom diameters are 30 and 50 inches; supposing it to be filled with liquid, and that it is then inclined till the remaining liquid just covers half the bottom, what is the quantity of the liquid left in the vessel?

Ans. 4766.666 inches.

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To find the solidity of the PARABOLIC UNGULAS *of a conic frustum:—*

Multiply the area of the base of the ungula by the greater diameter of the frustum, and divide the product by the difference of the diameters, and *reserve* the *quotient;* then multiply the difference of the diameters by the less diameter, and extract the square root of the product; multiply this root by four-thirds of the less diameter, and subtract the product from the *reserved quotient;* multiply this remainder by one-third of the altitude of the ungula, and the product will be the content.

147. Find the solidity of the parabolic ungula, cut from a conic frustum, the diameters of the ends being 15 and 9.6 inches, and its height 20 inches, the upper edge of the ungula terminating in the edge of the upper end of the frustum.

Ans. 446.2273 inches.

The versed sine of the base of the parabolic ungula is always equal to the difference of the diameters of the frustum; and hence, the versed sine of the base $=15-9.6=5.4$; and the area of the base equals 57.2738, and $\frac{57.2738\times15}{5.4}=$ 159.0941, the reserved quotient; and the $\sqrt{5.4\times9.6\times\frac{4}{3}}$ of $9.6=92.16$, which subtracted from $159.0941=66.9341$, and $66.9341\times\frac{20}{3}=446.2273$.

148. Let a vessel in the form of a conic frustum, the diameters of the bottom and top being 30 and 19.2 inches, be inclined so that its upper slant side shall be parallel to the horizon; required the quantity of fluid it is capable of containing in this position, the depth of the vessel being 20 inches.

Ans. 1784.9093 cubic inches.

149. Find the ungula which is complementary to that in the last example. *Ans.* 7873.603 inches.

When the angle DGB is greater than VAB, the section EDF is an *hyperbola,* and the ungula DEFB is an hyperbolic ungula.

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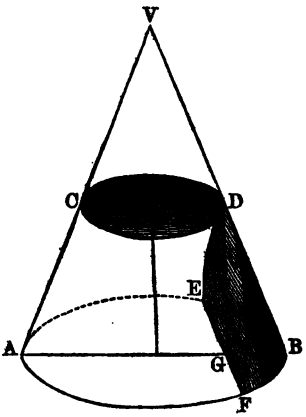
To find the solidity of an HYPERBOLIC UNGULA:—

HYPERBOLIC UNGULA.



Find the area of the segment of the base, (viz. EBF,) and multiply the said area by the greater diameter, (AB,) and *reserve* the product; find also the area of the hyperbolic segment, (EDF,) and multiply this area by the less diameter, (CD,) and multiply this product by the versed sine of the base, (GB,) and divide this product by GD, the abscis of the hyperbolic section, and subtract the quotient from the *reserved* product; multiply this remainder by one-third of the height, or altitude of the frustum, and divide the product by the difference of the diameters, and the quotient will be the content of the required ungula.

This rule will apply to any conic ungula.

The major axis of the hyperbolic section may be found by the following rule:—

Multiply the less diameter of the frustum by the abscis, (DG,) and divide the product by the difference between the greater diameter and the sum of the less diameter and versed sine of the base of the ungula, and the result will be the major axis.

To find the minor axis:—

Divide the versed sine of the base of the ungula by the difference between the greater diameter and the sum of the less diameter and versed sine of the base of the ungula, and extract the square root of the quotient; multiply this root by the less diameter, and the product will be the minor axis.

150. The diameters of a conic frustum are 30 and 20, and

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the abscis of the hyperbolic ungula, DG, is perpendicular to the base of the frustum, the altitude of which is 20 inches; required the major and minor axes of the hyperbolic section, and the solidity of the ungula.

Ans. 80 and 20 are the axes, and the area of the hyperbola EDF is found (by rule 12, ¶ 30) to be 285.6, nearly; and the content of the ungula, DEFB, is found to be 596.72.

151. Two men, A and B, are to receive $100 for digging a ditch; A agrees to dig for 87½ cents per rod, and B for $1.12½; when the ditch is dug, it is found that they have each earned $50; required the number of rods of ditch dug by each, and the rate at which they must be paid per rod, to satisfy the conditions of the question.

Ans. A digs 56¼ rods at 88⅝ cents, and B digs 43¾ rods at $1.14⅞ per rod.

152. The cylinder of a screw is 7 inches in diameter, and the distance between the adjacent threads is 12 inches; required the length of the spiral ridge of the thread of the screw, the perpendicular height being 120 inches.

Ans. 20.8767 feet.

153. The lengths of three right lines drawn from a certain point in an equilateral triangle to the three angles are 20, and 25, and 31 rods; required the side of the triangle.

Ans. 42.901 rods, nearly.

154. A jet of water is thrown upwards at an angle of elevation of 66 degrees, with a velocity of projection of 20 feet per second; required the *height* which it will attain, its *horizontal range*, and its *time of flight*, that is, the *time* required by each particle of water to pass through the parabolic curve.

Ans. 5.22, and 9.3 feet, and 1.14 second.

155. What figures can be so arranged about a point as to fill the entire space?

Ans. 6 trigons; 4 squares; or 3 hexagons.

156. It is required to cut a rectangular board 9 by 16

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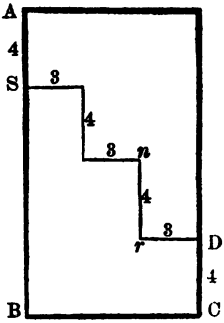
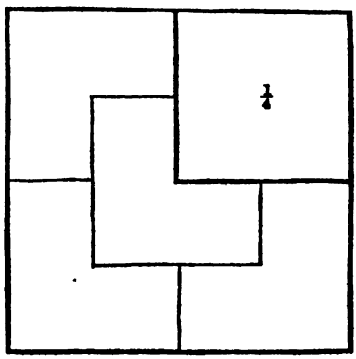
inches, so as to cover one square foot. Let ABC represent the board, AB being 16 and BC 9 inches. Four inches from A, at S, saw into the board 3 inches, parallel to BC; then saw downwards, parallel to BA, 4 inches, and then saw 3 inches parallel to BC, and so on to D, which is 4 inches from C. Then make DC correspond to or touch *nr*, and the board will be square, and 12 inches on a side; since 4 inches has been taken from the length, and 3 added to the breadth.



157. A gentleman has a farm exactly square; he lays out a square at one corner equal to one-fourth of the whole farm; he then wishes to divide the remainder into 4 equal pieces of the same form for his four sons; how may this be done?



The cut shows the manner in which the farm may be divided too clearly to need further explanation.

158. The mainmast of a ship is broken by a severe gale at sea; how may it be spliced without shortening it, and without using any timber except what is cut from the mast?

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From the part broken off at CD, cut out the piece D*sno*, say 7 feet long; then from the lower part CDB, cut out the piece *stc*D, 18 feet long, and slip it to the left till *s*D meets *no*, and it will break the joint, or splice the break; and the piece *no*D*s* will fill the vacancy *itce*, and make the mast complete.

¶ 82. GUAGE POINTS.

GAUGE POINTS FOR SUPERFICIES AND LENGTHS.

	Gauge Points on B.
For *Boards, in feet*	12. inches.
For the circumference of a circle	3.1416
" side of a square in a circle	0.7071
" side of an octagon	0.3833
" side of a trigon	0.866
" side of a pentagon	0.588
" side of a hexagon	0.5
" side of a heptagon	0.437
" side of a nonagon	0.337
" side of a decagon	0.309
" side of an undecagon	0.282
" side of a dodecagon	0.259
" side of an octagon in a square	0.414
" side of a square equal to a circle	0.8862
" side of a cube cut from a sphere	0.5773

GAUGE POINTS FOR AREAS.

	Gauge Points on D.
For a square, *in feet*	12.
" square, *in acres*	40. or 12.65
" circle, in feet	13.54

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	Gauge Points on D.
For a circle, in acres	14.28
" trigon, in feet	18.234
" trigon, in acres	19.23
" pentagon, in feet	9.147
" pentagon, in acres	9.644
" hexagon, in feet	7.452
" hexagon, in acres	7.861
" heptagon, in feet	6.26
" heptagon, in acres	6.635
" octagon, in feet	5.466
" octagon, in acres	5.760
" cycloid	0.65 hl
" cube, 6 on C to	1.000
" sphere	0.565
Or, 3.1416 on C to	1.000

GAUGE POINTS FOR SOLIDITIES.

	Gauge Points on D.
For square prisms	12. contents in ft.
For cylinders	13.54
For an equilateral triangular prism	18.234
" " pentagonal "	9.147
" " hexagonal "	7.452
" " heptagonal "	6.26
" " octagonal "	5.46
" " nonagonal "	4.824
" " decagonal "	4.32
" " undecagonal "	3.92
" " dodecagonal "	3.59
For a coal-pit	1.756
" square pyramid	20.78
" cone	23.453
" sphere	16.583
" parabolic spindle	9.89
" paraboloid	19.153
" coal-pit, in cords	19.87

For earth's curvature	1.325
" falling bodies	1.
" square root	1.
" horse-power of a steam-engine, 30 on C to	10.

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See *gauge points* for gallons, bushels, &c. ¶ 58; for the weight of bodies, ¶ 69; for rectangular and triangular prisms, ¶ 34 and 35; for mill logs, ¶ 37; and for levelling, ¶ 62.

GAUGE POINTS for the *Engineer's Sliding Rule* may be found under the description of the said rule, ¶ 1, and under Machinery, ¶ 79.

To gauge the weight of a pipe of cast-iron by the engineer's rule:—Set the length on B under 489 on A, and over the bore of the pipe found on D will be found the weight of the bore, supposing it were solid iron; then add twice the thickness of the pipe to the diameter of the bore, and the sum will be the outside diameter of the pipe; then over this diameter found on D, will be found the weight of the pipe, supposing it were all solid iron; from this weight subtract that previously found, and the remainder will be the weight of the pipe.

EXAMPLE.—What will be the weight of a pipe, 12 inches long, 8 inches bore, and $\frac{3}{4}$ inch thick? *Ans.* 64 pounds.

Set 12 on B to 489 on A, and against 8 on D are 157 lbs. on C; then twice $\frac{3}{4}$ added to 8=9.5 inches, the external diameter; then over 9.5, found on D, will be found 221 lbs. on C; and 221−157=64, the answer.

When pipes have flanges on the ends, every two flanges may be reckoned equal to one foot of pipe.

THE END.

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